Monetary Policy in a Small Open Partially Dollarized African Economy

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Abstract

The study uses a Dynamic Stochastic General Equilibrium to describe the conduct of monetary policy in a small, open, and partially dollarized Tanzanian economy. The structure of the model incorporates the expectations of agents and the dynamic relationships are explained in terms of structural representations that characterize the behaviour of the firm, household and central bank. The model incorporates several conditions that are consistent with most modern new Keynesian models in that it allows for a number of nominal and real rigidities. The parameters in the model are estimated with Bayesian techniques, after it has been applied to Tanzanian data. The effects of individual shocks, including those that may be used to describe the conduct of monetary policy, are then considered. These simulations suggest that despite the existence of partial dollarization in the Tanzanian economy, monetary policy has important, short-term, real effects.

1.1 Introduction

The central banks of most developed and many developing economies make use of structural macroeconometric models to assist with policy analysis and forecasting (Tovar, 2009). Some of the variants of these models follow Laxton et al. (2006), which describes the framework that is currently used by several central banks on the African continent.‡ The central features of this model incorporate various nominal and real rigidities, as well as a large number of shocks. Within this framework, the rational expectations of economic agents are accounted for and each of the equations in the model has a structural economic interpretation.

In the case of Tanzania, which may be classified as a small open economy, the analysis of monetary policy is largely conducted with the aid of reduced-form models, which do not take into account the expectations of agents or potentially important structural relationships that could exist between the variables. Another interesting feature of the Tanzanian economy is that most transactions may be conducted in either Tanzanian Shillings or United States Dollars. In addition, the agents in the Tanzanian economy may choose to hold dollars to store value during periods of abnormally high inflation. This behaviour gives rise to a partially dollarized economy, as discussed in Reinhart et al. (2014).

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† The model is usually termed the Quarterly Projection Model (QPM) and is the basis of comparison for many country economists of the International Monetary Fund. Over time, a large number of IMF Working Papers have extended the work of Laxton et al. (2006). See for example, Carabenciov et al. (2013), Blagreave et al. (2013), Freedman et al. (2009a), Freedman et al. (2009b), Carabenciovet al. (2008b), Carabenciovet al.(2008a), and Laxton et al. (2008).
Various authors have made use of different structural models to investigate the conduct of monetary policy in an economy that encounters partial dollarization. These include dynamic stochastic general equilibrium (DSGE) models which have been applied to several South American economies. Important recent contributions include the work of Castillo et al. (2013) who suggest that two forms of partial dollarization are present in the Peruvian economy (currency substitution and dollar-price indexation). In addition, Salas (2010) makes use of a structural macroeconometric model to suggest that the expectations channel has become more prominent in the transmission of monetary policy shocks.

The model in this paper follows that of Salas (2010), in which the foreign currency holdings affect the domestic aggregate demand equation. In addition, we also assume that the central bank may choose to intervene in the foreign exchange market, as rapid exchange rate depreciation could reduce the ability of the agents to repay foreign currency denominated debt. When we allow for financial dollarization (where dollars are preferred as a store of value) and currency mismatches, the balance sheet effects associated with large exchange rate swings are likely to emerge, which could be detrimental if the central bank does not intervene in the foreign exchange markets.

After the model has been log-linearized it is applied to quarterly Tanzanian data for the period, 2001q1 to 2013q3. The starting date of this sample represents the earliest available quarterly data point for Tanzanian output. The model includes measures for domestic output, inflation, real effective exchange rate, nominal currency depreciation and the nominal interest rate, as well as foreign output, inflation and the corresponding interest rate.

The parameters that pertain to the critical behavioural equations in the model are then estimated with Bayesian techniques. These parameter estimates suggest that transaction dollarization and dollar-price indexation are quite important, while financial dollarization is highly prevalent. When we then turn our attention to the impulse response functions, we note that although one could suggest that the existence of partial dollarization in the Tanzanian economy may result in ineffective monetary policy, the effect of a shock to the short-term domestic interest rate continues to result in important changes in the real variables.

In terms of the contribution of this paper, to the best of our knowledge this is the first application of a structural macroeconometric model for a partially dollarized African economy that has been used to investigate the effectiveness of monetary

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§ See also the references noted in Castillo et al. (2013) for a discussion of earlier applications of DSGE models with partial dollarization in South American economies.

**See, Calvo and Reinhart (2002), Reinhart and Reinhart (2008) and Reinhart and Rogoff (2009) for a discussion on the interventions by central banks in the foreign currency marketsin emerging market economies.
policy. The rest of the paper is organized as follows. Section 1.2 describes the methodology, section 1.3 provides details of the data and section 1.4 includes details of the parameter estimation techniques. The results are discussed in section 1.5 and the conclusion is contained in section 1.6.

1.2 Methodology

1.2.1 Theoretical DSGE Model

The structure describes the cyclical behaviour of a small open and partially dollarized economy, in a setting that is consistent with several DSGE models. It is a short-run model in the sense that the variables are expressed in terms of deviations from their equilibrium, or steady-state values. A number of new Keynesian features have also been included, in the form of nominal and real rigidities. We also include features that are consistent with agents that display rational expectations and backward-looking indexation.

In the tradition of Botman et al. (2007), while the model is consistent with many micro-founded models, it has not been derived from explicit microfoundations. It is argued that this practice is consistent with those followed by most central banks that are “engaged but not married to economic theory” Laxton, et al. (2006). In what follows, we show how the linear conditions in the model are related to purely microfounded model representations.††

The household and the aggregate demand expression:

The microfoundations for the household of most small open economy models, which follow that of Galí and Monacelli (2005) and Justiniano and Preston (2010) suggest that the representative household seeks to maximize utility in the function,‡‡

$$E_t \sum_{\tau=0}^{\infty} \beta^\tau \left[ \varepsilon \left( \frac{C_t - \zeta C_{t-1}}{1 - \sigma} - \frac{N_t^{1+\gamma}}{1 + \gamma} \right) \right]$$

where the variable $C_t$ refers to consumption and $N_t$ to labour, both in period $t$. The parameter $\beta$ is the time-discount factor, $1/\sigma$ is the intertemporal-elasticity of substitution, $1/\gamma$ is the Frisch-elasticity of labour supply, and $\zeta$ is a consumption-habit parameter. The exogenous demand shock is then represented by $\eta_t$.

The budget constraint of the household may then be expressed as,

†† Of course it would be possible to derive microfoundations for most of the final model equations. However, such a time-consuming procedure would be of little use in this instance.

‡‡ Similar models have been used to describe monetary policy in other African countries. See for example, Steinbach et al. (2009) and Alpanda et al. (2010a, 2010a; and 2011). For a more elaborate discussion of the microfoundations that have been employed in this exposition, see Alpanda et al. (2010a).
where \( P_{ct} \) is the price of the consumption good, \( B_t \) is the domestic bond, \( E_t \) is the nominal exchange rate, \( B^*_t \) is the foreign bonds that are held by domestic residents, \( W_t \) is the nominal wage rate, \( R_t \) is the nominal domestic interest rate, \( R^*_t \) is the nominal foreign interest rate, \( \Pi_t \) is profits received from the domestic intermediate goods producers, and \( P_t^p \) is the aggregate price index that is used to deflate the above measures.

In an open-economy setting the consumption and price indices refer to a combination of goods that are produced by domestic and foreign manufacturers, such that

\[
C_t = \left(1 - \alpha\right) P_{ht}^{\eta-1} C_{ht}^\eta + \alpha P_{ft}^{\eta-1} C_{ft}^\eta
\]

and

\[
P_{ct} = \left(1 - \alpha\right) P_t^{1-\eta} + \alpha P_{ft}^{1-\eta}
\]

where \( C_{ht} \) and \( P_t \) refer to the domestic consumption and price indices, while \( C_{ft} \) and \( P_{ft} \) refer to the foreign counterparts. The importance of foreign goods in overall consumption is represented by \( \alpha \) and \( \eta > 0 \) is the elasticity of substitution between domestic and foreign goods.

After imposing the constraint for no Ponzi schemes and where we assume that the goods market clears, when consumption is equal to output (i.e. \( C_t = Y_t \)), one is able to derive the following microfounded equilibrium expression for aggregate demand,

\[
y_t = \frac{1}{1+\zeta} E_t \left[ y_{t+1} \right] + \frac{\zeta}{1+\zeta} y_{t-1} - \frac{1-\zeta}{\sigma(1+\zeta)} \left( r_t - E_t \left[ \pi_{t+1} \right] \right) - \epsilon_t
\]

where the use of small letters denotes that the variable is expressed in terms of the logarithmic deviation from steady-state values. This expression may be termed the new Keynesian IS-curve for a small open-economy.

In the model for the small open-economy with partial dollarization, we make use of the following expression that describes the aggregate demand dynamics,
where after making a comparison with equation (5), we note that 
\( \alpha^{re} \approx 1/(1 + \zeta) \), 
\( \alpha^y \approx \zeta/1 + \zeta^* \), and 
\( \alpha^{mc} A^r \approx (1 - \zeta)/\sigma(1 + \zeta) \). In the final expression, the \( A^r \)
parameter refers to the proportion of monetary holdings that are denominated in
the domestic currency. In addition to the variables from the microfounded
aggregate demand expression, we have augmented this equation in (6) with
information relating to foreign interest rates, \( r_t^* \), the change in the terms of trade,
tot, the real effective exchange rate \( q_t \), the fiscal impulse, \( fis_t \), and the measure
for foreign aggregate demand, \( y_t^* \).

The motivation for including these additional terms should be self-evident, where
in a partially dollarized economy the interest rate on holdings of dollars would be
of importance. Note that in this case, the \( A^{mc} \) parameter refers to the proportion
of domestic monetary holdings that is denominated in dollars. Furthermore, an
improvement in the terms of trade (the price of exports relative to the price of
imports) should improve aggregate demand in the domestic economy. In addition,
depreciation in the real effective exchange rate (which is represented by an
increase in \( q_t \)) would also improve conditions for exporting additional goods, while
a positive fiscal impulse (as represented by an increase in government expenditure
that is over and above the trend) would stimulate domestic demand. Similarly, an
increase in the global (foreign) economic output would also contribute towards more
positive trading conditions, which would provide an impetus for an increase in
domestic output.

The firm and the aggregate supply expression:
In the new Keynesian model we assume that the differentiated goods of the
monopolistic competitive intermediate producers are indexed by the continuum
\( j \in [0, 1] \), such that the final good may be expressed with the aid of the following
aggregation function,

\[
Y_t = \left[ \int_{0}^{1} \frac{\partial \xi}{\partial \theta} \left( Y_{j,t} \right) \frac{\partial Y_{j,t}}{\partial Y_{j-1,t}} dj \right]
\]  

(7)

where \( \theta_t \) is the elasticity of substitution between the intermediate goods and
\( \xi_t = \theta_t / (\theta_t - 1) \) may be used to describe the gross mark up over marginal costs.
The representative intermediate firm would then set prices to maximize the present value of profits. Since future earnings are discounted at the same rate as the household, their objective function may be expressed as

$$\max E_t \sum_{\tau=0}^{\infty} \beta^{\tau-t} \frac{1}{\lambda_{\tau}} \left[ \frac{P_{j,\tau}}{P_z} Y_{j,\tau} - \frac{W_{z}}{P_z} N_{j,\tau} - \frac{\kappa}{2} \left( \frac{P_{j,\tau}}{P_{j,\tau-1}} \right)^2 Y_{\tau} \right]$$

(8)

where the last term is the quadratic cost of price adjustment as described in Rotemberg (1982). The parameter $\kappa$ regulates the magnitude of the price adjustment costs, which are also scaled by aggregate domestic output. The price-adjustment cost is incurred when the increase in the firm’s own price deviates from the past inflation rate, where the parameter $\phi$ regulates the extent to which current price changes are indexed to past inflation.

These expressions may be used to drive the familiar new Keynesian Phillips curve with indexation,

$$\pi_t = \frac{\beta}{1+\beta \phi} E_t [\pi_{t+1}] + \frac{\phi}{1+\beta \phi} \pi_{t-1} + \frac{\theta - 1}{\kappa (1+\beta \phi)} mc_t + \frac{\theta_t}{\theta_t - 1}$$

(9)

where the use of small letters denotes that the variables are expressed in terms of logarithmic deviations from their steady-state values. In addition, a complete expression for marginal costs, $mc_t$, in a microfounded small open-economy may take the form of,

$$mc_t = \gamma y_t + \alpha (tot_t) + \frac{\sigma}{1-\zeta} (c_t - \zeta c_{t-1})$$

(10)

The aggregate supply condition in the model that we use for the partially dollarized economy may be expressed as,

$$\pi_t^* = (1-b^m) \left( b^s \pi_{t-1} + (1-b^s) \right) + b^s y_{t-1} + b^m \left( \pi_{t+m}^* + \Delta q_t \right) + e_t^*$$

(11)

where $\pi_t^*$ refers to core inflation (that part of the inflationary process that is not subject to transitory shocks) and $\pi_t^m$ refers to imported inflation. The term $\Delta q_t$ refers to the steady-state of the real effective exchange rate in first difference. When comparing the above expression with that of the microfounded model, we note that $\beta/(1+\beta \phi) \approx (1-b^s)^2 \left( 1-b^p \right)$ and $\phi/(1+\beta \phi) \approx (1-b^s)^2 \left( 1-b^p \right)$. In addition, it is also worth noting that the marginal costs in the microfounded model are largely influenced by the deviations of output from its steady-state values, and as a result a measure of the output gap has been included in the model for the partially dollarized economy.
The remaining term in the microfounded marginal cost expression relates to the effects of changes in the terms of trade, which is a measure for the price of exports in terms of the price of imports. This is the source for which we may incorporate imported inflation into the microfounded model. In the model for the partially dollarized economy we make use of a more elaborate expression for imported inflation, which takes the form of,

\[ \pi_t^m = c^p \pi_{t-1}^m + c^{ef} \left( \pi_t^* + 4.\Delta s_t \right) + \left( 1 - c^p - c^{ef} \right) \left( \pi_{t-1}^m + 4.\Delta s_{t-1} \right) + \epsilon_t^m \]  

where \( \pi_t^* \) is a measure of foreign inflation, which is expressed in terms of domestic currency units after accounting for the annual change in the nominal exchange rate, \( \Delta s_t \). Imported inflation is also influenced by past changes in the price of imported raw materials and other intermediate goods, \( \pi_t^m \), which is expressed in terms of domestic currency after including a term for the annual depreciation of the domestic currency. The shock to imported inflation is given by \( \epsilon_t^m \) and the contemporaneous quarterly exchange rate pass-through would be represented by the product of the coefficients \( b^{p^*} \) and \( c^{p_{ef}} \).

**Modified uncovered interest rate parity:**

As in the microfounded model that makes use of the interest rate parity condition to close the open economy features in a model, we make use of the following uncovered interest rate parity condition

\[ 4(s_t^e - s_t) = i_t - i_t^* - r_{p_t} + \epsilon_t^s \]  

Where \( r_{p_t} \) refers to the risk-premium. As an alternative, one could make use of complete risk-sharing conditions to close off the open economy features of the model, however, Alpanda et al.(2010b) suggest that the use of the modified interest rate parity condition that incorporates a risk-premium may result in more consistent explanation of the data, when applied to an African economy. The stochastic term \( \epsilon_t^s \) represents the disturbance to the modified uncovered interest rate parity condition.

In the model for the partially dollarized economy, exchange rate expectations, \( s_t^e \), are described by the weighted average of backward-looking and forward-looking components, such that

\[ s_{t+1}^e = (1-\theta)s_{t+1} + \theta \left( s_{t-1} + 0.5 \left( \Delta \bar{p}_t + \bar{\pi}_t - \pi_t^* \right) \right) + \epsilon_t^e \]  

where \( \epsilon_t^e \) is the stochastic error term.
The monetary policy rule:
Various expressions for the monetary policy rule have been used to describe the way in which the central bank sets short-term nominal interest rates. The ease with which one is able to accomplish this objective is facilitated by the fact that the majority of monetary policy rules that have been considered are linear by definition; and most are based on those that are described in Taylor (1993). The rule that is in our model for the partially dollarized economy takes the form

\[ i_t = f^i i_{t-1} + (1 - f^i) \left( \tau_i + f^r \left( E_t \left[ \pi_{t+4}^a \right] - \pi_t \right) + f^y \frac{y_t + y_{t-1}}{2} + f^s s_t \right) + \xi_t \]  

(14)

where \( i_t \) refers to the nominal domestic interest rate, \( \tau_i \) refers to the steady-state or natural interest rate, \( E_t \left[ \pi_{t+4}^a \right] \) refers to the expected value of the one-year-ahead value for annual core inflation, and \( \pi_t \) is the central bank annual inflation target. Contractionary monetary policy innovations would then be effected through a positive shock to the \( \xi_t \) stochastic term.

In terms of the coefficients, \( f^i \) would refer to the extent of interest rate smoothing, \( f^r \) refers to the central bank reaction function to deviations in expected inflation from the target rate, \( f^y \) refers to the central bank reaction function to deviations in the average of the past two quarters output gap and \( f^s \) refers to the central bank response to changes in the nominal depreciation rate of the domestic currency.

Additional model equations that define the equilibrium conditions:
In addition to the six essential behavioural equations (5, 6, 11, 12, 13 and 14) that have been described above, the model includes an additional forty-three equations that are used to define the equilibrium conditions. These include those that are used to convert quarterly to annual measures, definitions for steady-state values, and those that define the persistence in variables and innovations. A complete list of the model equations is contained in the appendix D.

1.3 Data
The dataset makes use of quarterly data that extends over the period 2001q1 to 2013q3. The start date of the sample is the earliest date for which a measure of quarterly output is available. A total of eight observed variables are used to reflect measures of: Domestic output growth, \( y_t \), consumer inflation, \( \pi_t \), nominal interest rate, \( i_t \), real effective exchange rate, \( q_t \), nominal currency depreciation, \( s_t \), foreign output growth, \( y_t^* \), foreign inflation, \( \pi_t^* \), and foreign nominal interest.

\#Gali (2008) considers the implications of a few of these in small open economy setting.
Most of the data for the Tanzanian economy was obtained from the IMF’s International Financial Statistics (IFS) database, with the exception of the nominal interest rate data, which were obtained from the Bank of Tanzania. The data for the United States economy was obtained from the Federal Reserve System.

A seasonal filter was applied to the unseasonally adjusted measure of domestic output, which is used to derive a measure of the domestic output gap. The filter used is described in (Hodrick and Prescott, 1997). The seasonally adjusted measure of output was used for the foreign output gap, which was also derived from a Hodrick-Prescott filter. Domestic consumer inflation rates are expressed as the year-on-year logarithmic difference in the average quarterly consumer price index, while foreign inflation is derived from the deflator. The official interest rate in Tanzania is transformed to an annualised rate to reflect the nominal domestic interest rate, while the annualised Federal Funds rate is used for the foreign interest rate. The logarithmic difference of the nominal exchange rate between the Tanzania Shilling and US dollar has been used to measure nominal exchange rate depreciation, while similar transformations have been applied to the real effective exchange rate.

### 1.4 Estimation Techniques

The dataset includes eight variables that span only a 12-year period. To alleviate potential problems relating to insufficient degrees of freedom, Bayesian techniques are used to estimate the parameter values. The use of these methods to estimate the parameter estimates is recommended by several researchers, including Fernández-Villaverde (2010), Fernández-Villaverde et al. (2010) and Del Negro and Schorfheide (2011).

Given the size of the model, we have elected to calibrate all of the parameters that do not relate to the essential behavioural equations. The values that we have used for this exercise follow those that are contained in Laxton et al. (2006) and Salas (2010), and may be found in the appendix D. For the parameters in the behavioural equations we have provided details of priors in Tables 6.1 to 6.5, which also include the posterior values of these parameters.

### 1.5 Results

#### 1.5.1 Posterior Estimates

All of the parameters in the aggregate demand equation were estimated, with the exception of the coefficient that is attached to the previous observed measure of the output gap, which was calibrated to the valueits persistence in a first order autoregressive model (see appendix). The results that are contained in Table 1.1 suggest that the forward-looking aspect in aggregate demand is slightly smaller than the backward-looking element, which was calibrated at 0.55. In addition, given the weights of $A^r$ and $A^{rs}$ the posterior estimate for $a^{rmc}$ would suggest that the weight on the real interest rate gap in domestic currency is almost 11%,
while the foreign currency equivalent is approximately 5%. It is also worth noting that the coefficient value for the influence of the fiscal impulse is relatively large, while the posterior values for the other coefficients are relatively small.

When we consider the results from the aggregate supply equation in Table 1.2, we note that the agents in the model are largely backward-looking, as the current value of inflation is largely influenced by past values of this variable. After multiplying out the respective parameters the backward-looking coefficient is associated with a value of 0.74, while the forward-looking component is associated with a value of 0.22. In addition, we note that the posterior value for the parameter that is related to the measure of output is 0.11, which is slightly higher than the prior value. In terms of the effect of the sources of imported inflation, only a small proportion (7%) may be attributed to the effect of raw materials, while the largest component is due to inflation in the foreign country.

When considering the factors that influence exchange rate expectations, we note that the agents in the model are more likely to make use of forward-looking expectations, as oppose to backward-looking adaptive expectations. Evidence of this is provided by the decline in the value of the posterior estimate for \( \theta \), from 0.5 to 0.4.
The parameters for the monetary policy rule are contained in Table 1.4, where we note that all the posterior values are relatively closely associated with their priors, which could be used to infer that they are consistent with international evidence. In this case the response of the central bank to inflation is considerably stronger than the response to any movement in the output gap.

The posterior values for the parameters that pertain to the shocks and the respective persistence in these shocks are contained in the appendix.

1.5.2 Impulse Response Functions
The results of the Bayesian impulse response functions are displayed in Figures 1.1 through 1.4. The confidence interval for each of these functions is set to 90%, which ensures that they are relatively broad and consistent with the intervals that are reported for the posterior estimates.

Figure 1.1 provides details of the simulated effects of a monetary policy shock that result from a 1% increase in the standard deviation in $\epsilon_t$. We note that output and inflation decline, where the effect on output is greater than the effect on inflation, which is largely consistent with economic theory. We also note that the nominal exchange rate strengthens following the initial impact period, which is consistent with the results of a number of other studies (a decline in rate of currency depreciation is equivalent to appreciation in the exchange rate).

The impulse response function that describes the effects of a shock to domestic aggregate demand are displayed in Figure 1.2, where positive innovation to aggregate demand results in an increase in output, which fuels inflation and causes the interest rate to rise. The rise in the interest rate causes output to stabilize and also provides some impetus for renewed currency strength.
Figure 1.1: The Impulse Response Function - Monetary Policy Shock

Figure 1.2: The Impulse Response Function - Aggregate Demand Shock
The effects of a cost-push shock, which affects the aggregate supply relationship, are displayed in Figure 1.3. In this case, a positive innovation to the cost-push shock causes an increase in inflation and a decline in output. From a theoretical perspective, such a shock would shift the Phillips curve and presents a less favourable trade-off between inflation and output. The rising in the rate of inflation also results in an increase in interest rates, since the monetary policy rule places more emphasis on rising inflation than declining output. The short-term rise in interest rates is associated with a certain degree of currency strength. However, after the effects of the spike in inflation are realised, the external value of the currency depreciates.

Figure 1.3: The Impulse Response Function - Cost-Push Shock

Figure 1.4 displays the effects of a shock to the exchange rate, where we note that a negative shock, which results in an appreciation of the nominal exchange rate, is accompanied by a decline in output, as the terms of trade will deteriorate. The decline in aggregate demand would result in a decline in the rate of inflation, which would allow for the central bank to ease monetary policy conditions.
1.6 Conclusion

This paper considers the use of a small macroeconometric model for the Tanzanian economy. While all of the relationships in this model have not been derived from explicit microfoundations, it shares a number of features that are consistent with purely theoretical models. In addition, the model incorporates several conditions that are consistent with most modern new Keynesian models in that it allows for a number of nominal and real rigidities. The setting is also consistent with that of other models for a small, open-economy that employs an interest rate sharing condition with a risk premium, which is used to close off the open-economy features in the model. The agents in the model are also able to engage in the type of behaviour that is frequently observed in partially dollarized economies, where households and firms may choose to hold and transact with a foreign currency. The results suggest that after including a role for partial dollarization in the model, monetary policy continues to have short-run effects on the real variables through the traditional interest rate channel of the transmission mechanism. In addition, we also observe that the effects of aggregate demand, cost-push, and exchange rate shocks are consistent with economic theory, where the response of interest rates would appear to be pragmatic.
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References


Appendix

Model Equations

\[ y_i = a^1 y_{i-1} + a^{m1} y_{i+1} + a^{m2} Q_{i-1} + a^{tot} \left( A^{tot} \text{tot}_i + \left( 1 - A^{tot} \right) \text{tot}_{i-1} \right) + \ldots \]

\[ a^1 q_i + a^1 \text{fis}_i + a^{m1} y^*_i + \varepsilon_i \]

\[ \pi^* = b^{mp} \left( \pi^m - \Delta q_{ss} \right) + (1 - b^{mp}) \left( b^p \pi^p_{i-1} + (1 - b^p) \pi^p_{i+1} \right) + y^p_i + \varepsilon^p_i \]

\[ 4 \left( s^e_i - s_i \right) = i_i - i^*_i - rp_i + \varepsilon^e_i \]

\[ i_i = f^i i_{i+1} + (1 - f^i) \left( \frac{\bar{i}_i + f^p \bar{\pi}_i + f^p \left( y^e_i + y^p_{i-1} \right) + f^p \Delta s_i}{2} \right) + \varepsilon^e_i \]

\[ \pi^m = c^p \pi^m_{i+1} + c^{pf} \left( 4 \Delta s_i + \pi^p_{i+1} \right) + (1 - c^p - c^{pf} ) \left( 4 \Delta s_{i+1} + \pi^m_{i+1} \right) + \varepsilon^m_i \]

\[ i^*_i = \left( 1 - \theta^p \right) \left( \bar{r}^e_i + \pi^p_{i+1} \right) + \theta^p \bar{i}_{i-1} + \varepsilon^e_i \]

\[ Q_i = - \left( A^pr_i + A^{m1} i^*_i \right) \]

\[ r_i = r_{i+1} - \bar{r}_i \]

\[ r_{i+1} = i_i^4 - \pi^4_{i+1} \]

\[ i_i^4 = \varepsilon_i^{bp} + 0.25(i_i + i_{i+1} + i_{i+2} + i_{i+3}) \]

\[ \bar{r}_i^* = r_{i+1}^* - \bar{r}_i^* \]

\[ r_{i+1}^* = s^4_{i+1} - s_i + i^4_{i+1} - \pi^4_{i+1} \]

\[ i^4_{i+1} = \varepsilon_i^{p^p} + 0.25(i_i^4 + i_{i+1} + i_{i+2} + i_{i+3}) \]

\[ s^e_{i+1} = (1 - \theta) s_{i+1} + \theta (s_{i-1} + 0.5 (\Delta q_{ss} + \bar{\pi}_s - \pi^*_{ss})) ) + \varepsilon^e_i \]

\[ s^e_{i+1} = \varepsilon^e_i + \omega (s_{i-1} + (\Delta q_{ss} + \bar{\pi}_s - \pi^*_{ss}) 1.25) + (1 - \omega) s_{i+1} \]

\[ \Delta s_i = s_i - s_{i-1} \]

\[ \tau_p = \theta^p \tau_{p+1} + (1 - \theta^p) \tau_{p+1} \]

\[ \pi^*_{i+1} = 0.25 \left( \pi^*_i + \pi^*_{i+1} + \pi^*_{i-2} + \pi^*_{i-3} \right) \]

\[ q_i = \Delta s_i + q_{i+1} + 0.25 \left( \pi^*_i - \pi^* - \Delta \bar{q}_i \right) \]

\[ \pi^m_{i+1} = \pi^m_{i+1} \theta^m + \pi^*_{ss} (1 - \theta^m) + \varepsilon^m_i \]

\[ \pi^*_i = \pi^*_{ss} (1 - \theta^m) + \theta^m \pi^*_{i+1} + \varepsilon^*_i \]

\[ \bar{i}_i = \pi^*_i \chi + (1 - \chi) \pi^*_i \]
\[ \pi_{1}^{nc} = \pi_{s} s (1 - \delta) + \delta \pi_{1}^{nc} + \epsilon_{i}^{nc} \]
\[ \pi_{t} = \pi_{t}^{nc} - \pi_{s} s \]
\[ \epsilon_{i} = (1 - \delta) (\pi_{s} s + \pi_{s} \epsilon_{i-1}) + \delta \pi_{t-1} \]
\[ \text{tot} = \text{tot}_{t} \delta + \epsilon_{t}^{\text{tot}} \]
\[ \text{fis}_{t} = \delta \text{fis}_{t-1} + \epsilon_{t}^{\text{fis}} \]
\[ y_{t} = y_{t-1} \delta + \epsilon_{t}^{y} \]
\[ \Delta q = \Delta q_{s} (1 - \delta) + \delta \Delta q_{t-1} \]
\[ \tilde{r}_{t} = (1 - \delta) (\tilde{r}_{s} + \phi_{s}^{i}) + \delta \tilde{r}_{t-1} \]
\[ \tilde{r}_{t}^{*} = (1 - \delta) (\tilde{r}_{s}^{*} + \phi_{s}^{A^{*}}) + \delta \tilde{r}_{t-1}^{*} \]
\[ \epsilon_{i}^{y} = \rho^{y} \epsilon_{i-1}^{y} + \xi_{i}^{y} \]
\[ \epsilon_{i}^{\pi} = \rho^{\pi} \epsilon_{i-1}^{\pi} + \xi_{i}^{\pi} \]
\[ \epsilon_{i}^{z} = \rho^{z} \epsilon_{i-1}^{z} + \xi_{i}^{z} \]
\[ \epsilon_{i}^{i} = \rho^{i} \epsilon_{i-1}^{i} + \xi_{i}^{i} \]
\[ \epsilon_{i}^{m} = \rho^{m} \epsilon_{i-1}^{m} + \xi_{i}^{m} \]
\[ \epsilon_{i}^{s} = \rho^{s} \epsilon_{i-1}^{s} + \xi_{i}^{s} \]
\[ \epsilon_{i}^{iy} = \phi_{i}^{y} (1 - \rho^{iy}) + \rho^{iy} \epsilon_{i-1}^{iy} + \xi_{i}^{iy} \]
\[ \epsilon_{i}^{iy*} = \phi_{i}^{A^{*}} (1 - \rho^{iy*}) + \rho^{iy*} \epsilon_{i-1}^{iy*} + \xi_{i}^{iy*} \]
\[ \epsilon_{i}^{e} = \rho^{e} \epsilon_{i-1}^{e} + \xi_{i}^{e} \]
\[ \epsilon_{i}^{ae} = \rho^{ae} \epsilon_{i-1}^{ae} + \xi_{i}^{ae} \]
\[ \epsilon_{i}^{em} = \rho^{em} \epsilon_{i-1}^{em} + \xi_{i}^{em} \]
\[ \epsilon_{i}^{ae*} = \rho^{ae*} \epsilon_{i-1}^{ae*} + \xi_{i}^{ae*} \]
\[ \epsilon_{i}^{nc} = \rho^{nc} \epsilon_{i-1}^{nc} + \xi_{i}^{nc} \]
\[ \epsilon_{i}^{tot} = \rho^{tot} \epsilon_{i-1}^{tot} + \xi_{i}^{tot} \]
\[ \epsilon_{i}^{fis} = \rho^{fis} \epsilon_{i-1}^{fis} + \xi_{i}^{fis} \]
\[ \epsilon_{i}^{ys} = \rho^{ys} \epsilon_{i-1}^{ys} + \xi_{i}^{ys} \]
\[ y_{t}^{\text{data}} = y_{t} + \mu_{t}^{y} \]
\[ \pi_{t}^{c, \text{data}} = \pi_{t} + \mu_{t} \]
### Calibrated Parameters

**D 1 : Parameters - Calibrated Values**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta^0$</td>
<td>0.9</td>
<td>$\bar{r}_{ss}^*$</td>
<td>2.5</td>
</tr>
<tr>
<td>$A^r$</td>
<td>0.3</td>
<td>$\pi_{ss}^*$</td>
<td>2</td>
</tr>
<tr>
<td>$A^{rs}$</td>
<td>0.15</td>
<td>$\pi_{ss}$</td>
<td>2</td>
</tr>
<tr>
<td>$W$</td>
<td>0.8</td>
<td>$\bar{p}_{ss}$</td>
<td>1</td>
</tr>
<tr>
<td>$\beta^x$</td>
<td>0.25</td>
<td>$\bar{r}_{ss}$</td>
<td>$\bar{r}<em>{ss}^*$ + $\bar{p}</em>{ss}$</td>
</tr>
<tr>
<td>$\gamma_p$</td>
<td>0.7</td>
<td>$\phi_{ss}^4$</td>
<td>8</td>
</tr>
<tr>
<td>$\gamma_{rm}$</td>
<td>0.7</td>
<td>$\phi_{ss}^{A,*}$</td>
<td>4.5</td>
</tr>
<tr>
<td>$\chi$</td>
<td>0.6</td>
<td>$\rho^i$</td>
<td>0</td>
</tr>
<tr>
<td>$\gamma_{nc}$</td>
<td>0.4</td>
<td>$\rho^n$</td>
<td>0</td>
</tr>
<tr>
<td>$\gamma_{i}^l$</td>
<td>0.5</td>
<td>$\rho^l$</td>
<td>0.6</td>
</tr>
<tr>
<td>$\gamma_{tot}$</td>
<td>0.8</td>
<td>$\rho^{lp}$</td>
<td>0.95</td>
</tr>
<tr>
<td>$\gamma_{ys}$</td>
<td>0.5</td>
<td>$\rho^{yl}$</td>
<td>0.95</td>
</tr>
<tr>
<td>$\gamma_{ys}$</td>
<td>0.9</td>
<td>$\rho^{ys}$</td>
<td>0</td>
</tr>
<tr>
<td>$\gamma_{ys}$</td>
<td>0.9</td>
<td>$\rho^{ys}$</td>
<td>0</td>
</tr>
<tr>
<td>$\gamma_{ys}$</td>
<td>0.95</td>
<td>$\rho^{ys}$</td>
<td>0</td>
</tr>
<tr>
<td>$\gamma_{ys}$</td>
<td>0.95</td>
<td>$\rho^{ys}$</td>
<td>0</td>
</tr>
</tbody>
</table>

| $\bar{q}$ | 0   | $\rho^{ys}$ | 0     |
Parameter Estimates for Shocks

### D 2: Parameters - Persistence in Shocks

<table>
<thead>
<tr>
<th>Parameter Estimate</th>
<th>Distribution</th>
<th>Prior Mean</th>
<th>Std. Dev.</th>
<th>Mode</th>
<th>Posterior Mean [10% 90%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho^y$</td>
<td>beta</td>
<td>0.5</td>
<td>0.1</td>
<td>0.483</td>
<td>0.314 0.645</td>
</tr>
<tr>
<td>$\rho^z$</td>
<td>beta</td>
<td>0.15</td>
<td>0.05</td>
<td>0.165</td>
<td>0.088 0.252</td>
</tr>
<tr>
<td>$\rho^s$</td>
<td>beta</td>
<td>0.15</td>
<td>0.05</td>
<td>0.147</td>
<td>0.066 0.206</td>
</tr>
<tr>
<td>$\rho^e$</td>
<td>beta</td>
<td>0.3</td>
<td>0.1</td>
<td>0.29</td>
<td>0.125 0.404</td>
</tr>
<tr>
<td>$\rho^{4,e}$</td>
<td>beta</td>
<td>0.6</td>
<td>0.1</td>
<td>0.612</td>
<td>0.471 0.756</td>
</tr>
<tr>
<td>$\rho^{tot}$</td>
<td>beta</td>
<td>0.4</td>
<td>0.1</td>
<td>0.407</td>
<td>0.259 0.557</td>
</tr>
</tbody>
</table>

### D 3: Parameters - Standard Deviation of Shocks

<table>
<thead>
<tr>
<th>Parameter Estimate</th>
<th>Distribution</th>
<th>Prior Mean</th>
<th>Std. Dev.</th>
<th>Mode</th>
<th>Posterior Mean [10% 90%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\zeta^y_t$</td>
<td>invg</td>
<td>0.4</td>
<td>4</td>
<td>0.321</td>
<td>0.13 0.512</td>
</tr>
<tr>
<td>$\zeta^z_t$</td>
<td>invg</td>
<td>0.4</td>
<td>4</td>
<td>0.55</td>
<td>0.117 1.076</td>
</tr>
<tr>
<td>$\zeta^s_t$</td>
<td>invg</td>
<td>0.4</td>
<td>4</td>
<td>0.373</td>
<td>0.191 0.527</td>
</tr>
<tr>
<td>$\zeta^e_t$</td>
<td>invg</td>
<td>0.4</td>
<td>4</td>
<td>0.278</td>
<td>0.143 0.447</td>
</tr>
<tr>
<td>$\zeta^{4,e}_t$</td>
<td>invg</td>
<td>0.4</td>
<td>4</td>
<td>0.471</td>
<td>0.111 1.034</td>
</tr>
<tr>
<td>$\zeta^s_t$</td>
<td>invg</td>
<td>0.4</td>
<td>4</td>
<td>1.189</td>
<td>0.86 1.471</td>
</tr>
<tr>
<td>$\zeta^{4*}_t$</td>
<td>invg</td>
<td>0.4</td>
<td>4</td>
<td>0.231</td>
<td>0.18 0.271</td>
</tr>
<tr>
<td>$\zeta^{y*}_t$</td>
<td>invg</td>
<td>0.4</td>
<td>4</td>
<td>0.548</td>
<td>0.465 0.623</td>
</tr>
</tbody>
</table>