BUSINESS MANAGEMENT REVIEW
2025 vol 28, No. 1
https://doi.org/10.56279/bmrj.v28i1.7011

Research Article

BMR, 28,1

89

Double entry bookkeeping: Its philosophical and mathematical underpinnings

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Abstract

This paper explores double entry bookkeeping as a theoretical and practical framework for the accounting discipline. Three foundational theoretical underpinnings for double entry bookkeeping are provided which constitute scientific and philosophical justification for the practice. These underpinnings consist of two philosophical stances and one mathematical position. In regard to the two philosophical stances, double entry bookkeeping is considered to be based on a figurative language perspective as well as a causality perspective. Regarding the mathematical position, the paper argues that double entry bookkeeping is grounded in an algebraic perspective specifically Pacioli group. The paper, hence, contributes to the understanding of double entry bookkeeping. The main contribution of this paper is to advance analysis of double entry bookkeeping while also subjecting this framework, which is widely applied, to critical review. Double entry bookkeeping becomes more applicable as the organisational complexity increases. This is why the double entry bookkeeping's application at the micro-accounting level is minimal, while at the macro-accounting level its application is wide. The paper contributes to the literature on accounting theory by demonstrating that double entry bookkeeping is more than a technique by showing that it is theoretically grounded on a solid philosophical and mathematical foundation for the purpose of constructing and communicating economic reality.

Keywords: *Double entry bookkeeping (DEB), micro-accounting, macro-accounting, causality, figurative language, mathematical modelling, Pacioli group*.

Received: 26 January 2025
Revised: 12 June 2025
Accepted: 16 June 2025
Published: 09 July 2025

Citation: Chalu, H. (2025). Double entry bookkeeping: its philosophical and mathematical underpinnings. *Business Management Review,* ***28*(1), 89-110**. https://doi.org/10.56279/bmrj.v28i1.7011

Introduction

This paper explores the conceptualization of double entry bookkeeping (DEB) in accounting and accounting-related literature.[1] Indeed, most accounting textbooks dealing with DEB consider it casually as merely a technique for recording transactions. As such, these textbooks fail to provide a scientific grounding for DEB's significance and usage (Zhou & Lamberton, 2021). Montgomery (1938) observed that accounting writers have been preoccupied with business transactions and the recording process but not theory. This reduces the DEB's analytical contribution to accounting science to just record keeping. It could be argued that DEB has been considered mainly as a tool for account-keeping and not for account-witting, which is more scientific and it calls for deeper analysis if we use Galagan's perspective as cited by Kuter et al., (2019). More recent scholars have been questioning the relevance of DEB in elementary accounting courses (Ingram, 1998; Pincus, 1997; Vangermeersch, 1997). This situation is considered to have risen because of limited understanding of the theoretical richness embedded in DEB (Aho, 2005; Gleeson-White, 2011; Sombart, 1967[1913]).

Various authors have explored the importance of DEB (Batiz-Lazo, Borreguero, Maixe-Attes & Torrado, 2012; Carlin, 2019; Khansalar & Kashefi-Pour, 2020; Wirth & Mattessich, 2006; Yamey, 1947). They have argued that DEB is a cornerstone of modern accounting, providing devices to help people and organisations to pursue their goals rationally and objectively, facilitating the growth of capitalism, and contributing to economic development as well as structuring and influencing how organisations work. According to Littleton (1966 [1933]), as cited by Aho, 2005), DEB contributed to an evolution of accounting from bookkeeping fictions to scientific facts. Aho (2005, p. xiii) went on stating that:

> [T]he distinguishing equation of DEB acknowledges an existential truth, evidently it was not formulated in writing until early fourteenth century in Italy. This being the case, the circumstances surrounding its written expression constitute fascinating problem in the sociology of knowledge, as it turns out, in the sociology of modern consciousness!

The German philosopher Werner Sombart considered the DEB's invention to be of similar importance to the discoveries of great scientists such as Galileo, Harvey and Newton and he considers DEB to embed ideas of gravitation, circulation of blood and conservation of matter (Most, 1976). This view is consistent with the earlier perspective of Cayley (1894), who praised DEB and compared it with Euclid's theory of ratios, considering it to be absolutely perfect.[2] Likewise, Sprague (1908) pointed to the richness of DEB's philosophical foundations, arguing that "as a branch of mathematical and classificatory science, the principles of accountancy may be determined by a priori reasoning and do not depend upon customs and traditions which surround the art". However, Sprague (1908) was not specific on philosophical assumptions and theories relevant for DEB.

While Sprague (1908) emphasized the importance of theoretical foundations and scientific reasoning, stating that these parameters provide the basis of DEB, this is contrary to the presentation of DEB in accounting textbooks as merely a record keeping technique. In most of the accounting textbooks, DEB is treated in a rudimentary fashion, reduced to a mere calculative formulation to guide the preparation of accounts. Textbooks' overlook of DEB's significance and foundational assumptions led Van Cleve (1913) to argue that the problem of DEB is its failure to delve theoretically into the meaning of debit and credit. Journal articles, for their part, tend to focus on the historical aspects of DEB, treating it as a dead subject. This overlooks the DEB's solid theoretical foundations while also ignoring its wider value in today's economic environment. Indeed, the use of DEB can contribute practically and intellectually to both the conceptualization of accounting and economic thinking.[3] As such, its full significance, main assumptions and appropriateness of this innovation have been inadequately exposed.

The current paper takes up Van Cleve's criticism and looks into the meaning of these two terms i.e. debit and credit within DEB grounded on a theoretical perspective. Hence, the main question addressed in this paper is: what are the theoretical assumptions of DEB? As such, this paper addresses a longstanding gap on how DEB has been conceptualized. It dwells on theoretical perspectives on DEB drawing on philosophical and mathematical premises. Specifically, this paper considers two philosophical stances and one mathematical position. In regard to the two philosophical stances, DEB is determined to be based on a figurative language perspective as well as a causality perspective. Regarding the mathematical position, the paper argues that DEB is grounded on an algebraic perspective. These philosophical and mathematical assumptions enable DEB as an accounting conceptualization to grasp economic activities carried out by economic entities from household level to national government level and assist in 'big picture' economic thinking.

Overall, this paper makes three contributions to the extant literature. The first is its use of three theoretical perspectives grounded on philosophical and mathematical assumptions – namely, figurative language, causality and mathematical modelling – to generate a broader perspective on accounting science (or 'account-witting' as per Galagan, 1961 as cited by Kuter et al, 2019). As such, it expands beyond the limited perspective of financial reporting, which is concerned with the communication of product of accounting science (i.e. production of financial statements). In other words, financial reporting is limited to the 'art' side of accounting and not the scientific side of it. This perspective is consistent with Van Cleve (1913, p. v):

> [O]ne of the strangest things in the history of arts and science is that this great system of accounting which by reason of its compactness and convenience, has come into universal use, should have attained so high degree of development on the practical side, while on the theoretical side it is and always has been in a state of confusion.

Martinelli (1974), too, observed that historians had failed to develop a general theory on the early evolution of accounting. Hence, this paper attempts to add to the general theory of DEB, because while figurative language may be linked to financial reporting, financial reporting hardly takes into consideration the remaining two perspectives, namely, causality and mathematical modelling. Van Cleve (1913) found that textbooks covering financial reporting (in the name of accounting) failed to provide even a remote conceptualization of the principles on which the art of financial reporting was based. In our view, the DEB's theoretical basis could help in designing the accounting system and also inform the mechanics of accounting, including how data are processed or manipulated to produce accounting information (Sterling, 1970).[4]

The second contribution lies in the recognition of DEB as a multi-theoretical concept, hence extending the coverage of DEB in the literature hence, helping us to understand its conceptualization and how it advances economic thinking. For example, while most previous studies in accounting look at DEB from a historical angle, studies in statistics and economics dealing with macro-variables such as national income consider DEB as a useful approach or model up to now (see Gleeson-White, 2011). This has made DEB more appreciated in other disciplines other than accounting (Demski et al., 2006). Using a multi-paradigm perspective helps to expose the DEB's current usefulness and importance, without losing sight of its historical nature. This broader understanding will enable accountants and other practitioners to appreciate the DEB's influence on organisations' operations and remove it from 'accounting wildernesses.

Third contribution is based on mathematical modelling, whereby it is shown that among the mathematical assumptions which can be used to explain DEB, algebra is more appropriate. As such this paper attempts to explain in simple terms group algebra specifically Pacioli group which has been advanced and derived by Ellerman (2014) and Rambaud et al., (2010). These authors have based their works from classical mathematicians such as Cayley (1894), DeMorgan (1869), Hamilton (1831) as well as Littlewood (2002[191949]) on the group of differences. While these mathematical authors have attempted to link with accounting, the connection is difficult and complex hence most of accounting textbooks and literature have not even mentioned them. Hence, it is not surprising to algebraic operations on ordered pairs are largely unknown in accounting (Ellerman, 2014). This paper while it is not dwelling on mathematical derivation, but considers that including it as one of the theoretical assumptions, will expose these formulations which will help in further exploration. As such it is considered that further and deeper investigation will serve to deepen the understanding of DEB's usefulness and engendering greater appreciation for the role of scientific accounting. In this sense, the paper builds on the efforts of Bryer (1993), who argued that to explain DEB, there is a need to understand the social, economic and political context in which it is applied. Furthermore, this exploration will help to overcome the literature's current emphasis on the historical. Indeed, DEB has been applied for almost a thousand years (Martinelli, 1974), and there is no sign of it fading away – as attested to by many examples in the banking world, monetary policy crafting and in national accounts systems (see Beretta & Cencini, 2020; Gleeson-White, 2011; Menezes et al., 2020; Murai, 2024).

To achieve the above mentioned contributions, this paper is further organised as follows. The next section presents an overview of DEB covering the practice's origin and significance. Sections three and four present the two philosophical stances underpinning DEB, respectively, figurative language and causality. Section five then presents the mathematical modelling perspective of DEB. As such, sections three to five basically provide arguments on the extent that DEB is theoretically and scientifically grounded. Section six concludes the paper by summarizing its findings, contributions and limitations.

DEB: An overview

Most accounting textbooks define DEB as a logical system for recording transactions that reflect the dual nature of each transaction. In this system, every debit entry must have a corresponding credit entry, and vice versa. This implies that different accounts (at least two) are affected by each transaction (Kieso et al., 2012; Pratt, 2011; Standfield, 2002; Weygandt et al., 2014). Most textbooks, however, neglect to adequately define 'debit' and 'credit', sufficing to present them simply as the left and right side of accounts. These textbooks' failure to provide a clear meaning for these two terms is, in our view, due to a narrow, 'technical' view of DEB, to use Canning's (1929) terminology.

The technical view is important because it helps us to understand the mechanics of accounting and ensures that the advantages of DEB are achieved. These advantages include its self-correcting nature (Palepu et al., 2013), equality of debits and credits (Pratt, 2011; Weygandt et al., 2014), and arithmetic accuracy of the books of account. Moreover, it establishes a foundation for the accrual accounting perspective5 and emphasizes dynamic accounting, in contrast to static accounting[6]. On the issue of the equality of debits and credits, DEB provides explanations to the analogy of equal observations made by internal and external observers and this equivalence is made at least by two observers having different viewpoints (Birkhoff & Neumann, 1936). DEB's accuracy is linked to its self-correcting mechanism and is achieved with a quick check of account entries. In short, all accounts with a debit balance summed should equal the sum of all accounts with credit balance. Recognizing the advantages of DEB, Goethe (1824, p.28, as translated by Thomas Carlyle) marvelled:

> What a thing it is to see the order which prevails throughout his business. By means of this he can at any time needing to perplex himself in details. What advantages does he derive from the system of bookkeeping by double entry? It is among the finest inventions of human kind.

In extension, Suzuki (2003a, 2003b), considering the relation between accounting and economics, argued that DEB made it possible to analyse the internal structure of the accounting entity in terms of both balance sheet and profit and loss. Sombart (1925, p. 254, as cited by Martinelli, 1974, p. 243), observed that DEB's advantages created a systematic way of analysing transactions and helped to generate logical connections between various inputs:

> The history of double entry bookkeeping must begin with the motto "At the origin there was the account, the ratio". Even today the bookkeeping doctrine is rightly called the name of "doctrine of accounts" and similarly the French and Italian languages use this word to indicate bookkeeping as a whole; *comptabilite ragioneria*. But what has validity for the complete system has even a greater validity for the first beginning of the method. From accounts came systematic bookkeeping through the classification made by accounts, the gathering of information was analysed for the first time and into the chaos a logical connection was introduced by which any greater endeavour could lay its foundation.

Despite these advantages, concentration on the technical aspects reduces DEB to a mere mechanical item without origin and deep theoretical foundations (see Martinelli, 1974; Sangster, 2016). For example, Martinelli (1974) provided four fundamental conditions that need to be satisfied when applying the DEB method. These conditions were derived from Fabio Besta and include the following.[7] First is to have accounts which have laterally divided sections. Second is constant application of the same monetary unit. Third condition is constant reference for each entry to its cross-entry which is a recurring element used to link or identify specific transactions, accounts or journal entries for the purpose of maintaining consistency and accuracy of accounting records. Fourth, two complete sets of anti-thetical accounts must be present, one called elementary or patrimonial accounts and another set comprising derived or income accounts. These fundamental conditions alluded by Martinelli (1974) indicate the importance of accounts in the origin of DEB. This was similarly recognized by Sombart (1925, p. 118-119) as follows:

> Double entry bookkeeping is bourne of the same spirit as the system of Galileo and Newton… With the same means as these, it orders the phenomenon into an elegant system, and it may be called built on the basis of mechanistic thought. Double entry bookkeeping discloses to us the cosmos of economic World by the same method as later, the cosmos of the stellar universe was unveiled by the great investigation of natural philosophy…. One can scarcely conceive capitalism without double entry bookkeeping, they are related as are form and context. It is difficult to decide; however, whether in double entry bookkeeping capitalism provided itself with a tool to make it more effective or whether capitalism derives from the spirit of double entry bookkeeping.

DEB is also considered to have brought about the birth of modern accounting (Gleeson-White, 2011; Littleton, 1966[1933]; Parker, 2013[1984]). According to Gleeson-White (2011), Most (1972) and Soll (2014), the invention of DEB was vital to the development of capitalistic enterprises because it permitted the full representation of the flow of capital through business. It was through DEB that the conditions for capital formulation were possible, as it facilitated a focus on wealth creation.[8] According to Soll (2014), founders of modern economic thought considered DEB essential to the development of successful economies and modern capitalism. Still on the importance of DEB, Lin (1992) argued that DEB remains the core of computerization in the era of information revolution. For example, it is considered that electronic spreadsheets which are computer applications for computation, organization, analysis and storage of data in tabular form were designed based on DEB perspective hence attracted greater attention from accountants (Bradbard et al., 2014). Likewise, today's technologies such as Blockchain are also based on DEB. It is this flexibility of DEB which has made it useful not only for recording financial transactions but also for financial accountability matters. As argued by Soll (2014) that any political stability of countries is grounded in the culture of accountability. This accountability culture relies on DEB, not only because it provides a method to calculate profit but also because it brings about the concept of 'balanced books', which could be used to judge and hold accountable a political administration.

Based on this broad based importance of DEB, it is important to have a broader conceptualization of this invention if we had to use Sombart's words. The conceptualization of DEB can be explained using quantum logic as provided by Abramsky (2020), Birkhoff and von Neumann (1936) as well as Khrennikov (2007). According to this perspective, the observation of physical items or systems can be done by two eyes with different or opposing point of view. In other words, the two eyes observe the World together with the rest of the World. Using example of a chair as a physical system, the left eye sees it as a chair while on the other hand, the right eye sees there is no chair. As such if these eyes are observed, they are like false friends in the sense that they look similar but they are not because their viewpoints about the chair are different. Each eye has its own T-account [following the statistical theory of tabulation by Watkins (1915)]. However, these T-accounts are labeled different to consider the opposing views whereby the left eye gives a defined real observation of the world and the right eye is given an imaginary point of view of the World because it focuses on what is not. As such it can be argued that the two eyes present real and imaginary point of view of the World which resemble an item and its reflection. The observation of an item could be considered to be an internal view while its reflection could be considered to be an external viewpoint, whereby the left eye represents the internal viewpoint and right eye represent the external viewpoint.

The above analogy also applies to DEB since it provides an eye of the World when it comes to business transactions meaning that there are two eyes recording the same observations (transactions). However, this recording should be in the opposite direction but equal implying equivalence perspective. Under the equivalence perspective, debit must equate to credit, and vice versa. Thus, assets are written from an internal viewpoint (i.e., from the firm's viewpoint) and liabilities are written from an external viewpoint (i.e., from external entities' and shareholders' point of view). As such, liabilities are considered broadly to comprise both external and internal components. The internal component of liabilities is called 'capital' or 'equity' because it belongs to the shareholders. However, for the entity theory (which is the origin of separate entity principle) capital indicates a liability of a firm to its owners. The worldview of economic reality here implies that reality of economic events is not a neutral thing since is a subject of creation and influence of the observers (Hines, 1988). This emphasizes the point that accounting as philosophical discipline do not consider truth and reality of economic events as an objective issue since the numbers presented are based on facts and estimates, the latter being a function of different assumptions. As such accounting perspective is consistent with theories of truth, including the correspondence theory of truth (CTT), the coherence theory of truth and the pragmatic theory of truth (Walker, 2017).

The definitions and the importance of DEB beg for the answer about the evolution of this highly loaded perspective. Accounting literature has discussed widely about the birth and the growth of DEB (Aho, 2005; Bryer, 1993; Martinelli, 1974; Sangster & Rossi, 2018; Soll, 2014; Yamey, 1947). The exact date or time on when DEB became operational is still a debate among accounting historians. The debate on one side might have been caused by the publication of 'S*umma de arithmetica, geometria, proportioni et proportionalita'* (translated in English as Summary of arithmetic, geometry, proportions and proportionality) by Luca Pacioli in 1494. In this Treatise Luca Pacioli provided 11th Treatise of Section Nine called Particularis de Computis et Scripturis meaning *The rules of Double Entry Bookkeeping* (as translated by John B. Geijbeek in 1914). The publication of Luca Pacioli's work popularized DEB which led even some scholars to consider him as a father of accounting by crediting him with DEB invention.[9] Another reason could be the popularization of DEB by the Medici Bank. According to Parks (2005), Medici Bank was among the first to use debits and credits at the time when innovations had given the Italians monopoly of European finance.[10] Additional reason according to Soll (2014) is the use of Arab numerals[11] which made possible for fractions as well as minimize internal errors built in the Roman numerals. An increase in complexity of economic activities and trade required a complex accounting approach. The existing one which was using Roman numerals could not have achieved it (Soll, 2014, p. 9):

> All these account books and rolls beg the question: Did they at least work well? Surely a good diligent accountant, keeping daily records, should have been able to ascertain a certain level of mastery over accounts. This was the case in cash and inventory management, but even here it could not be exact. Without Arab numerals and therefore fraction, there was an internal error built in Roman numeral system. No matter how tenacious an account keeper was, the plethora of Xs, Ls and Is made cumbersome numbers such as DCCCXCIII (893) and left no space for fraction. New numbers and a new method of financial accounting were needed if complex trade was to flourish and advance.

This popularization of DEB has blurred the specific time at which this invention came to the business community. Different authors of both accounting and history have attempted to identify the period at which this DEB emerged. According to Gleeson-White (2011, p. 8):

> The rise and metamorphosis of double-entry bookkeeping is one of history's best-kept secrets and most important untold tales. Why? First, because it arguably made possible the wealth and cultural efflorescence that was the Renaissance. Second, because it enabled capitalism to flourish, so changing the economies of the world forever. Third, because over several centuries it grew into a sophisticated system of numbers which in the twenty-first century governs the global economy. This medieval artefact is still in daily use around the world.

To Aho (2005), DEB first appeared in Italian Cities and was documented in the Massari accounts of Genoa dated 1327 to avert frauds in the banks. Soll (2014) considers that DEB emerged in Tuscany and Northern Italy around 1300 and marked the beginning of history of capitalism and modern politics. Another reason for DEB emergence according to Soll (2014), is that accountants started to see accounting not only as a measure of holdings but also a way to calculate and distribute equity. Soll (2014) collaborates with Aho (2005) by providing ledgers of stewards of Genoa which provides that DEB was made an official in 1327 by the law called "*About Ledgers to Be Kept after the Manner of Banks*". This law required that business transactions to be recorded by two official accountants and auditors from city government consistent with internal and external view. Another study which dwelt on the originality of DEB is that by Peragallo (1938) which provided the emergence of DEB in Italian Cities of Genoa, Florence and Venice during the time of renaissance. Peragallo (1938) found that DEB appeared in Genoa around 1300 particularly in 1327 and states that:

> This system of double entry probably dates back to 1327 when many reforms were introduced in the Genoese government. At that time, because of many frauds, it was decreed that the ledgers were to be kept after the manner of banks (p.4). The fact that the government of Genoa issued a law that its bookkeeping records be kept in the manner of these *banchi* might indicate that double entry was used at the fairs. But no record exists by which the origin of double entry can be traced definitely to this source (p.17).

To support his claim that DEB predates 1340, Peragallo (1938, p. 3) states this:

> The most ancient double entry books known to exist are those of the Massari of the Commune of Genoa dating from the year 1340. These books are written in perfect double entry form which indicates that the system must have been in general use many years before.

In case of Florence, Peragallo (1938) argues that the emergence of DEB seemed to be before that of Genoa and he provides evidence of ledger of Peruzzi which preceded those of Massari by five years. While for Venice, Peragallo (1938) found that the existing documents of DEB were that of 1406. In this perspective, DEB emerged later in Venice compared to Florence and Genoa even though sometimes is called Venetian method. On the other hand, Martinelli (1974) considers that DEB may have started much earlier from the time of Roman Empire based on the criticisms of Plinio Bariola who considered that ancient Romans had reached high level of development in terms of law, public administration, commerce, trade and industrial institutions. Hence, it was considered logical to think that Romans could have knowledge of a balanced system of entries (Martinelli, 1974). Bariola states that (as cited by Martinelli, 1974, p. 188):

> Why should we consider ungrounded statements of those writers who assert that double entry bookkeeping was originated by Romans? Their hypothesis could be as groundless as the opposite theory. But there is a difference: while the former is based on a natural evidence of continuity or reproductions in the customs and institutions of ancient Roman civilization and the customs and organizations of the Middle Ages, the latter denies just for the sake of it, but it never supplies a convincing proof.

The statement of Bariola emphasizes that DEB was a result of growth of various institutions and society which demanded complex accounting systems. In other words, the growth and continuity in the areas of business, economics, government and others are the sources of origin of DEB. In this aspect, the emergence of DEB is considered to be a gradual process and could be considered as serendipity (see Basu & Waymire, 2024; Williams, 1978).[12] Basu and Waymire (2024) argue that DEB emerged from unplanned and spontaneous evolutionary process that led to many uses that were initially unforeseen. As such, it was not accepted instantaneously and universally. As argued by Martinelli (1974) that DEB did not appear immediately and in all circumstances more efficient than ancient techniques, hence most businessmen continued with their traditional accounting procedures while disregarding innovative techniques provided by DEB. The concluding point here is based on the arguments provided by Peragallo (1938) that no particular locality may have exclusive claim of being the birthplace of DEB because it was not conceived as whole but gradually as a result of efforts of different generations.

DEB and figurative language

Before relating figurative language to DEB, it is crucial to discuss it along language perspective. As argued by Dancygier and Sweetser (2014), figurative language is one aspect of what gives a text, in particular a poetic text, a special aesthetic value. Meaning that it helps to convey the message more beautifully than using literal language and provide different meanings which are richer and broader than literal translation (Dancygier & Sweetser, 2014). Aristotle recognized the importance of figurative language and he considered it to be the most significant feature of poetic composition and claimed that the greatest thing by far is to be the master of it. This is consistent to what was explained by Giora and Fein (1999) as well as Roberts and Kreuz (1994). Figurative languages can be divided into several types and include simile, metaphor, hyperbole, personification, synecdoche, onomatopoeia, understatement, irony, idioms, allegory, apostrophe and so on (Montgomery et al., 2007; Roberts & Kreuz, 1994). It is considered that figurative language is usually used to convey more complicated meaning or heightened effect. Roberts and Kreuz (1994) found that figurative language is associated with specific goal taxonomy which include humorous, emphasizing a certain issue, provoking thinking, clarification and so on. Their results (Roberts & Kreuz, 1994) suggest that specific discourse goals can be accomplished using a specific figure of speech.

Philosophically figurative language is connected with understanding the world and communicating abstract ideas. As such it is not just a stylistic choice of communicating rather than a tool for thinking and making meanings out of complex issues that is a tool for intelligibility (Paul, 1970). As argued by Perpich (2005), figurative language hence involves two orders, order of ontology which entails with the way in which things are given to consciousness and the order of ethics meaning the way in which human beings are

encountered. Willson-Quayle (1991; 1996) argues that most great philosopher have used figurative language to advance their views on particular issue hence figurative language forms the basis of political thinking. This is achieved through elements of figurative language which include metaphors and personification. According to Sala-Suszyńska (2016), using Aristotle definition, a metaphor is about giving the thing a name that belongs to something else, and this transference can be either from genius to species or vice versa. But it is also understood as interpretation or conceptualisation of one entity in terms of something else. It is considered as specific mental mapping and a form of neural coactivation that influences how people think, reason and imagine in everyday life (ibid.). On the other hand, personification is involved with giving human attributes and qualities to nonhuman or inanimate objects. In our view, personification may be considered as a subset of metaphors because it deals with giving human characteristics (a thing that belong to human to nonhuman (something else). As argued by Willson-Quayle (1996), there are two schools of thought for metaphors, first treating metaphors as linguistic confusion of names; and second regarding metaphors as giving rise to conceptions and mental images. Connecting with DEB it could be argued that it deals with the second school of thought and links with personification.

Paton (1917) for example, seems to support use of figurative language when it comes to personal accounts and rejects it for non-personal accounts which create confusion on whether Paton (1917) considered figurative language as a tool of thinking or he equated it with non-figurative language. In recognition of the important role of figurative language in accounting Amernic and Craig (2009) argue that accounting is a language-like discipline involving figurative expressions as well as elusive and perplexing modes of communication. Melissa Walters-York (1996) while identifies a number of accounting studies which recognize the importance of figurative language and apply it. considers that figurative language may serve to defamiliarize accounting practices and force people to reconceive the accounting practices contrary to the common presupposition that requires science and philosophy to be characterised by precision and absence of ambiguity which is the focus of literal languages. Amernic and Craig (2009) aimed at promoting critical conversation about the implication of metaphor in accounting, arguing that using accounting as an instrument has insidious, distortive and confounding outcomes because it encourages a belief that accounting is incapable of reporting other than representational faithfulness. Tucker (2017), on the other hand, was interested on the application of figurative language in teaching introductory accounting course to MBA students. In his teaching, he advocates the use of analogies, metaphors and similes to help students grasp properly accounting principles.

While the extant accounting literature accepts and recognizes the importance of figurative language, but it has not been considered as a philosophical stance to explain DEB. As a result, DEB is not clearly distinguished from single entry bookkeeping (SEB) and this can be observed from the confusion held by Paton (1917).[13] Aho (2005) provides evidence of rhetoric on DEB which include personalism in the sense that it attributes moral responsibilities to all components of business be it human or not. This tendency is consistent with personalistic theories of DEB as argued by Peragallo (1938), which assumes that all accounts represent persons contrary to Paton (1917) distinction. As such, the personalistic theories are linked to the personification type of figurative language. Personalistic account theory was developed contrary to positive or value account theory which considered that accounts should only deal with things. The use of personalistic theory was considered natural because ancient writers of accounting had the responsibility of teaching accounting to laymen; hence figurative language was used to facilitate the explanation (Peragallo, 1938; Tucker, 2017). Peragallo (1938) provides a number of ancient writers who had argued for personification in explaining DEB.

These writers include Ludovico Flori who published his work in 1633 and considered that all accounts were to be thought as real persons. Domenico Manzoni (1534) who grouped accounts into two groups live accounts for real persons such as debtors and dead accounts which were opened for things. Grouping accounts into personal and impersonal is also followed by Edmond Dégrange and Cinquecontisti School (*Five Accountists theory or French theory*) as well as Francesco Marchi who argued that debit him who he receives a value or who becomes a debtor for value and credit him who gives a value or becomes creditor for value. Giuseppe Cerboni who was in favour of distinguishing between real/living and dead accounts, came up with *Cerbonian Doctrine* or sometimes known as personalistic theory of accounts

(Esquerré, 1917) which equated right with credit and debit with duties and rejected the presence of dead accounts.

Peragallo (1938, p.99) provides an example of the use of figurative language under personification from Luca Pacioli's book:

> Paciolo used it frequently in the *Distinctio nona, Tractatus XI, De computis et scripturis* of his Summa. After correctly stating that the capital is the total resources of the proprietor and cash the total funds on hand, he went on to say that the proprietor assumes the position of creditor of his own capital. In chapter 23, he explained that a branch of a store is in effect the debtor of the proprietor, so that the latter may debit the store for all he puts in it and credit it for all he takes out of it, just as he would do in the case of a debtor who contracted a debt and subsequently paid it.

The above citation reinforces the figurative perspective hence indicating that to understand DEB, there is a need to consider nonliteral meaning of the terms debit and credit. In other words, debit and credit have meanings. This is contrary to Sprague's (1908) perspective who argued that debit and credit have no meaning and are just two sides of account. For example, Martinelli (1974) provides accounts which were kept from 1288 to 1290 which have entries beginning with *died* (I gave) and *ebbi* (I received). Gleeson-White (2011, p.173) recognizes this when she provides arguments by Bruce G. Carruthers and Wendy Nelson Espeland:

> In a 1991 paper, on accounting rhetoric Carruthers and Espeland argue that the symbolic language of double entry bookkeeping is as significant as its technical capabilities, a possibility not considered by Sombart and Weber. They argue that a double entry account is not just a piece of neutral information, but also an "account" or story: that accounting is not merely a technical practice, but also a means of framing a set of business transactions with rhetorical purpose.

The use of figurative language supports the argument provided by DeMorgan (1869) and van Cleve (1913) about the imaginary person. This is because one of the reasons for the growth or emergence of DEB as per De Roover (as cited by Martinelli, 1974) is to facilitate the mechanisms to keep records of their credits and their debits. As such, the accounting perspective has to deal with persons having debts. It is this angle that van Cleve (1913) argues that DEB is based on debt. Therefore, based on figurative language as our first philosophical assumption, it could be very easy to conceptualize DEB. This conceptualization considers that words debit and credit are used in accounts to convey meanings. If DEB uses words which mean nothing, then SEB would be considered the only rational system of accounting. Van Cleve (1913, pp.8-9) states:

> As long as bookkeepers speak of debiting Cash and crediting Merchandise to record a sale of merchandise for cash, they must admit that Merchandise is the imaginary person from whom we are supposed to borrow the merchandise which we give to the purchaser, and that Cash is the imaginary person to whom we are supposed to lend the money which we receive from the purchaser; or else they must admit that the language which they use consists of words without sense. If Cash means cash and Merchandise means merchandise, then to speak of debiting and crediting them is the height of absurdity.

DeMorgan (1869, pp. 181- 182) states that:

> The accounts are kept as if every different sort of account belonged to a separate person, and had an interest of its own, which every transaction either promotes or injures. If the student find that it helps him, he may imagine a clerk to every account: one to take charge of, and regulate, the actual cash; another for the bills which the house is to receive when due; another for those which it is to pay when due; another for the cloth (if the concern deal in cloth); another for the sugar (if it deal in sugar); one for every person who has an account with the house; one for the profits and losses; and so on.
>
> The account which is made debtor, or bound, is said to be debited; that which is made creditor, or released, is said to be credited. All who receive must be debited; all who give must be credited.

Van Cleve (1913, p.10) furthermore argues that what distinguishes DEB from SEB is the use of figurative language in the sense that SEB always uses literal language while DEB uses figurative (non-literal) language except when dealing with human beings or persons. In SEB, cash means cash, merchandize means merchandize and so on. However, for DEB, cash does not mean cash rather the imaginary person who owes the amount of cash. Merchandize does not mean merchandize; it means the imaginary person who owns the amount of merchandize

and so on. Based on this argument, it can be said that debit and credit have proper meaning of "*to receive*" and "*to give*", hence, DEB can be explained using Francesco Marchi's words *that debit him who he receives a value or who becomes debtor for value, credit him who gives a value or becomes creditor for value* (Peragallo, 1938). The appreciation of figurative language provides the meaning of words debit and credit. In this way, if an account is debited (meaning receiving), then another account is credited (meaning it is giving). This implies that giving causes receiving which is consistent with cause-effect relationship which is examined in the next section.

DEB and Causality assumption

Before we discuss causality assumption as a philosophical stance of DEB, we need to explore it as it relates to philosophy. This will help to reinforce the view that DEB is not only a technical thing but it is philosophically grounded. As argued by Peragallo (1938), consideration should not focus only on business transactions but also understanding philosophical stances because it is a good way of identifying the theoretical approach of DEB. This is crucial for scientific and logical thinking which create a foundation for accounting theory. The arguments held here are also consistent with the article published by Hatfield (1924) which argued and provided evidence of various philosophers and scientists who have helped to develop accounting including (apart from Luca Pacioli), Charles Morton, Grammateus or Schreiber, Jerome Cardan, Simon Stevin, Charles Hutton and so on.

Causality, which is still staple in contemporary philosophy sometimes is called causation or cause-effect, is concerned with one event, process, state or object that contributes to the production of another event, process, state or object (Bunge, 2017[1959]; Salmon, 1998; Illari& Russo, 2014). While this may look simple to explain or define, different philosophers over-time have struggled to operationalize the term causality. As observed by Bunge (2017[1959]), the subject of causality has produced diversity of views from flat rejection of causality categories to assertion of its coinciding with determination. For example, while Aristotle considered causality beyond materialistic perspective, other philosophers consider causality from metaphysics perspective as well as human creation perspective. In this aspect, others make a clear distinction between causation and explanation as argued by Lewis (1973) and Talbot (2021). These philosophers include David Hume, Immanuel Kant and so on. For example, to Aristotle, causality is linked to explanation of the answer to why question and it categorises causality into four main types namely material, formal, efficient and final. Kant, using the law of causality (that is the law of the connection of cause and effect), argues that causality and necessity go together in the sense that necessity produces effects (Kannisto, 2017). In other words, it can be argued that every event necessarily has just some cause or that some causes bring about the some effect. This is called every-event some-cause and some-cause-some-effect principles or weak causal principle (WCP) which considers that *every event has some cause* and strong causal principle (SCP) which considers that *every event belongs to a kind K and has a cause belonging to a kind L, such that necessarily every instance of L causes an event of kind K* (Hutton, 2021). David Hume considered causality to be concerned with the aspect of mind hence his definition of causality is grounded on regularity theory. In Hume's perspective, causality is considered to be a sequence of events. Hume (2007[1739], p. 169) states that:

> … a(n) object precedent and contiguous to another and where all objects resembling the former and plac'd in like relations of …

This perspective is consistent with Kant's law of causality which argues that necessarily, in every event there is something that is preceded and determined according to a rule by something else. In other words, every event involves a cause (Hutton, 2021; Kannisto, 2017; Kreines, 2009).

The causality perspective is concerned with the question: what kind of an event can be considered to be a cause and what kind of an event can be considered to be an effect? According to ontological perspective (Armstrong, 1996; Whitehead, 1929), the cause starts and the effect follows.

A is the cause and B is the effect
B is the cause and A is the effect

Action ⟶ Substance

However, in case of epistemological perspective (Maziarz, 2020; Talbot, 2021), causality concept is divided into regularity view (based on regularity theory by David Hume) and counterfactual view (based on counterfactual theory of causation by David Lewis). These two schools of thought extend approaches of causality to embrace regularity, probabilistic, counterfactual, mechanistic and manipulation. For example, the counterfactual view considers that:

> X causes Y, iff, without X, Y would not exist
> X causes Y, iff, the events are spatiatemporarily conjoined and X precedes Y

In this paper, we argue that despite different perspectives of causality, we think that the argument of properties of antecedents and contiguity or constant conjunction are crucial as far as accounting discipline is concerned. This perspective makes clear the difference between causation which is a metaphysical relation of events and the explanation which is an epistemological relation of events (relation is considered to be intelligible to us based on figurative language). As argued by Swanson (1993), accounting is a process through transactions which are economic actions across the boundaries of two higher-order living system. These transactions are components of economic exchange and should be reciprocal. The following examples can put clearly what we mean.

> **Example 1**
> *Philosophical point (based on necessary causes) 1*
> If X is a necessary cause of Y, then the presence of Y necessarily implies the prior occurrence of X. The presence of X, however, does not imply that Y will occur
> *Accounting point (based on necessary causes) 1*
> If Capital (C) is a necessary cause of Cash (Ca), then the presence of Ca necessarily implies the prior occurrence of C. The presence of C, however, does not imply that Ca will be available since the owner can produce capital in other form.
> **Example 2**
> *Philosophical point (assuming sufficient causes) 2*
> If X is a sufficient cause of Y, then the presence of X necessarily implies the subsequent occurrence of Y.
> *Accounting point (assuming sufficient causes) 2*
> If the sale of item (S) is a sufficient cause of asset [to bring Account Receivable (AR) or Cash (Ca)], then the presence of S necessarily implies the subsequent occurrence of AR or Ca.

Back to the DEB aspect, considering counterfactual theories, which define causality in terms of a counterfactual relation, the following definition of causality based on the notion of causal dependence by Lewis (1973) may be provided.[14]

> An event E causally depends on C iff, (i) if C has occurred, then E would have occurred, and (ii) if C had not occurred the E would not have occurred.

If we apply this perspective to DEB, assume the owner of the firm (be individual, group of individuals or government) introduces capital in the business in the form of cash. This cash once introduced, the personalised cash account will receive, hence being debited. Person in terms of capital account will be credited to signify giving as already argued in figurative language. In this case, therefore, cash account will be debited while capital account will be credited implying that debit only occurs if credit entry has occurred.[15] That is to say:

> An event D (Debit Entry) causally depends on event C (Credit Entry) iff, (i) if C (Credit Entry) has occurred, then D (Debit Entry) would have occurred, and (ii) if C (Credit Entry) had not occurred the D (Debit Entry) would not have occurred.

In addition, manipulation theories of causality are connected with accounting equation. According to Maziarz (2020), causality can be considered under manipulability whereby X causes Y if and only if a change in (intervention on) X modifies Y or manipulation of a cause will result in a case of the effect.[16]According to Collingwood (1940[2001]), the cause is something which is under human control and it is this control which can be used to influence the caused. In DEB, manipulation aspect of causality could be well grounded in accounting equation (particularly extended one) whereby a change in any element of the financial statements will influence the financial position of the entity.[17] But this perspective of certain element causing influence on a particular element only can be considered to be semi-causalism as argued by Bunge (2017[1959]).[18]

To conclude this section, the assumption of causality is the key foundation for DEB. Even though this is not clearly explained in the most of accounting literature, the main reason could be the term causality was not already invented at the time when DEB emerged. People look for causes to explain relationship. According to Basu and Waymire (2024), DEB was adopted for profit measurements based on forward-looking causal focus which was less emphasized in SEB. Basu and Waymire (2024) show that DEB construction was linked to causal exchange interactions. For example, costs are linked to revenue generation implying that costs cause revenue. But this may not be a sufficient condition because once products are produced; they need to be sold in order to generate revenue. And this is the essence of accrual accounting as argued by White (1937). In this case, probabilistic perspective may be appropriate in the sense that increasing costs in terms of producing more increases the likelihood of generating more revenues. Another accounting scholar to recognize causality assumption of DEB is Ijiri (1967; 1993). Ijiri (1967, pp. 102 – 105) provides the following explanation to support causation on DEB:

> What is the role of flow accounts in relation to stock accounts? We note that flow accounts, such as income accounts, are there to explain or 'account for' the reasons why stock account changed, either individually or in the aggregate. In the case of double-entry bookkeeping, what was this explanation by means of income statement accounts on why net assets (assets less liabilities) of the entity changed…

Furthermore, Ijiri (1993, p. 273) states:

> In the single-entry era, this "explanation" is what was missing. As mentioned earlier, merchants could figure out net income by comparing the two balance sheets, but they could not know why so much or so little income was earned because the books of accounts had only information that described "what" happened and not "why".

The causality perspective calls for mathematical modelling because mathematical model can help to represent and analyse causal relationship in a precise and quantitative way hence providing quantification effects, causal inference as predict nature of relationship. As such causality assumption of DEB, required a complementary assumption based on mathematical modelling which is covered in the following section. As already touched a bit about manipulation theory and accounting equation, DEB recognises the importance of mathematical formulation.

DEB and mathematical assumption

The last assumption of DEB considered in this paper is mathematical assumption which is based on mathematical modelling perspective. In mathematical modelling, a model is a simplified abstraction of reality. To build this abstraction of reality, the assumptions have to be made. According to Krawitz et al (2022), assumptions are necessary to solve problems, specify the missing information and help problem solvers to find a solution under the restrictive conditions. To Galbraith and Stillman (2001), assumptions provide building materials from the real World to bridge the divide between descriptive problem statement and its representation in mathematical terms hence are associated with model formulation, mathematical processing, as well as strategic choices in the solution process. In short, we can argue that assumptions are the basis of any mathematical model.

DEB as a mathematical model was developed between 12th and 13th century and it is based on a number of assumptions. However, since this model was invented almost one thousand years ago, its bases are and have been to the large extent implicit (Ellerman, 1985). There are several studies which have attempted to provide the mathematical assumptions of DEB. For example, Rambaud et al (2010) consider that presentation of DEB using T-accounts is based on T-diagram and argue that the T-diagrams list the debits and credits which form columns hence allowing the transactions to be processed in an ordered-pair form. This helps to produce the equation of this form (p.54):

$$X_{i1}+X_{i2}+X_{i3}+ \ldots\ldots+X_{iki}-Y_{i1}-Y_{i2}-Y_{i3-\ldots\ldots}- Y_{imi}$$

This equation can be expressed in Left Hand Side (LHS) and Right Hand Side (RHS) in order to make items non-negative. As such, an equal sign will be introduced and the equation will read as follows:

$$X_{i1}+X_{i2}+X_{i3}+ \ldots\ldots+X_{iki} = Y_{i1}+Y_{i2}+Y_{i3}+_{\ldots\ldots}+Y_{im}$$

Renes and Garst (2023), while recognising that DEB is first and foremost a system of recordkeeping, put a number of assumptions in terms of axioms and propositions. Of course, the emphasis on DEB being a recordkeeping system work, is in contrast with Ashton et al., (2004). While Ashton et al., (2004) were interested in the mathematical nature of DEB, they did not go into in-depth of the process of record keeping. They put an assumption of probability density function as well as linear combinations of log normally distributed variables of accounting identities which produce ratios. In other words, Ashton, et al., (2004) were concerned with the numbers which were already been produced by DEB (that's accounting model and not the mechanics of producing the numbers).

Renes and Garst (2023), therefore, attempted to provide assumptions on the mathematical aspects of DEB. The assumptions were based on design choice of DEB, generalization of DEB and comparing the recording system. In case of design choices, the assumption is made that DEB is dealing with an organisation which is company and that company has assets and liabilities whose values will be affected by economic events. In this perspective, they provided five axioms as indicated below (pp. 6-7):

> **Axiom 1** There exists some entity that contains n aspects with some value, with n accompanying accounts ai to record the value of these aspects, with $1 \leq n < \infty$.
> **Axiom 2** There exists a measurable set of economic events Ω with elements $\omega\tau$ that change the value of an aspect i of an organization at moment τ.
> **Axiom 3** The value (changes) of each aspect i is defined in the form of a real, signed measure $(\cdot):\Omega \rightarrow R$ that maps the events to a value in real numbers.4
> **Axiom 4** - All values are defined in the same unit: $\exists \mu s.t.\ \mu qi=1 \rightarrow ai \simeq an= 1\ \forall i$,
> - This common unit (the unit used in account n) is a monetary unit (Pure monetary unit assumption)
> **Axiom 5** In any valid bookkeeping statement with n accounts, the sum of all values is 0: $\Sigma(\cdot)=0ni=1\forall\omega\in\Omega$

The axioms provided by Renes and Garst (2023) prove that DEB is based on mathematical assumptions of aspects which are assets and liabilities of the entity and their values have to be recorded. Also the axioms show that DEB is concerned with economic events which need to be measured and recorded. Furthermore, since it deals with the value, then there is a need for monetary assumption. Lastly, the accounts have to be balanced to achieve what is called zero term in which the sum of debits equals the sum of credits. However, axioms 3 which deals with signed measurements and axiom 4 dealing with common unit which is monetary unit are not consistent with Ellerman (2014) perspective on mathematical grounding of DEB. As it could be seen later, Ellerman (2014) considers that DEB is grounded on mathematical model which use unsigned measurements (both numbers and components) and also not restricted to monetary measurement since the mathematical model allows for multi-dimensional approach.

Braun (2001), while concerned with mathematical models of assets and liabilities, did not consider the recording system, applied the moment of particles to analyse DEB.[19] This perspective can be considered to be based on *accountophysics* aspect in the sense that it combines accounting and physics.[20] Braun (2001) used Feynman-diagrams to describe DEB but as related with monetary systems. According to Melse (2008), Feynman-diagrams are graphs to perform scattering calculations in quantum field theory named after American physicist Richard Feynman, who introduced these diagrams in 1948. According to Harlander (2021), these diagrams do not directly represent the physical position or path of particles in spacetime instead they provide mathematical expressions that describe the probability of the extent of particle interactions. In another paper with Robert Fischer, Braun also translated DEB using momentum exchange of bouncing particles in space-time graphs. As such the paper (by Fischer & Braun, 2003b) provides illustration of momentum, force and energy of bookkeeping which in our view are consistent with transaction matrices as provided by Ellerman (2014), McGrail (1976) as well as Shank (1972). They (Fischer & Braun, 2003b) state that (p.4):

> Asset is positive particle momentum to the right of equally massed particles along one dimension. Liability is negative particle momentum to the left. Particles are further characterised by ownership i and currency c. The momentum P_{ic} is given in currency units. We display the particle momentum by trajectory arrows in space-time graphs. Ownership i is marked in the overhead and currency c by the collor of the arrow haft. Resting particles have no value and are not displayed. We call asset particles actons and liability particles passons. All bookkeeping information is contained in the graph.

Braun's papers based on physics-based assumptions help to provide mathematical assumptions grounded on physics. However, this might be possible if the flow is physical. Other studies which applied *accountophysics* approach to provide mathematical assumption are those conducted by Demski et al., (2009 and 2006). In these papers, DEB is connected with quantum perspective hence DEB is based on connectivity, prototyping, filtration and persistence as well as defects. These studies, however, have not clearly addressed the main two questions about mathematical assumptions of DEB. The first question is what is the meaning of "double" in DEB which is typically identified with the fact that two or more accounts are affected when the same characteristics of transactions are recorded? Second, why the DEB does not deal with negative numbers? These questions have been addressed by David Ellerman through a number of studies (see note 21 which covers the illustration as provided by Ellerman, 2014). Ellerman has reviewed a number of studies which have attempted to provide mathematical formulation. These studies include Cayley (1894), DeMorgan (1869) and Pacioli (2010[1494]). Based on these studies, Ellerman (1982; 2014) found that DEB is based on algebra category called Pacioli group and argue that is only algebra in which DEB does its calculation whether explicitly or implicitly. The algebra allows the DEB not only to perform calculation but also to use multiple measurement (multi-dimensions) units apart from monetary unit such as physical quantities (Ellerman, 2014; Ijiri, 1967). Pacioli group or group of differences in algebra is a specific type of algebra structure which deals with equations and systems that involve difference of values rather than derivatives like differential equations (Levin, 2008; Wibmer, 2021). It is used to study algebraic structures with an associated endomorphism denoted by σ which shifts or transform the elements of the structure. To Ellerman (1985) as well as Rambaud et al. (2010), Pacioli group are particular examples of the ordered-pairs construction using unsigned numbers.[21] Likewise, Sokolov and Bychkova (2004) considered that DEB is based on algebraic method. They justify their argument using Russian Scholars such as Sokolov and Bychkova (2004, p. 4), who state that: *the sum of material account balances is always equal to the algebraic sum of personal account balances.*

Based on the abstract algebra, the first question can be answered using the following formula:

LHS (Dr)		**RHS (Cr)**
(One account)		(Another account)
$X_{i1}+X_{i2}+X_{i3}+\ldots+X_{iki}$	=	$Y_{i1}+Y_{i2}+Y_{i3}+\ldots+Y_{imi}$

The two algebraic equations have to balance. As such, you cannot change one side without affecting the other. This brings the aspect of duality hence the "double" in the DEB (Ellerman, 2014) based on mathematical principle of duality because of two different points of view of looking at the same transaction (Artstein-Avidan & Milman, 2007; 2008; Atiyah, 2007; Wong, 2022). This perspective is consistent with Renes and Garst (2023) who argued that having this kind of a formula helps to ensure that accounting keeps tracks of all the stocks and all the flows. Likewise, Folsom (2023[1873]) argued that in keeping accounts by double entry, not only is this exchange recognised, but value in use or service also, making in fact two primary values whose trace is kept. This is consistent with earlier views provided by Most (2018[1970]) views that accounting, unlike auditing, does not deal with exclusion. It just deals with inclusion. The second question is about why DEB does not have a negative number? In this aspect the debit entries need to have credits that correspond and vice versa. In abstract algebra, positive and negative signs are eliminated because signed numbers are not used as provided by the properties of Pacioli group (see note 21). This approach allows for zero-term equation. In the same way, observing multiplicative properties, it could be observed that entries in debit side will have their reciprocal on the credit side of another account. Therefore, in a nutshell, we can argue that DEB is also mathematically grounded.

Conclusion

This paper aimed at addressing the theoretical foundation of DEB which is considered to be relevant based on two perspectives. First, DEB is widely used up to now despite being invented almost a thousand years ago. As such, since it is in a day-to-day business of the organisations and government in managing their economic affairs, it was considered that to understand its premises is crucial. Second, despite the technique being applied in accounting and economics textbook, it is less understood and most textbooks consider it as a mere 'technique'. To fill the knowledge gap on theoretical premises of DEB, this paper has provided a review of DEB background by providing examples of its applications.

At micro level also known as micro-accounting (here we mean corporate level because household accounting rarely uses DEB), DEB is widely used for accounting in terms of financial accounting and management accounting. It is the only technique which has been accepted to represent economic reality. At macro-level considered as macro-accounting, DEB is widely used in national accounts as well as in the formulation of monetary policies.[22] As such, it could be argued that DEB is also an accepted technique for national economic system. Hence, to address its theoretical premises, this paper provides three types of assumptions. The first two assumptions are based on philosophical stance and they are figurative language and causality perspectives. In case of figurative language, DEB is grounded on the perspective that accounts are persons hence debit and credit represent receiving and giving respectively. As such contrary to popular perspective, debit and credit have meanings and they are not just sides of accounts. Based on the figurative language which provides assumption of personification, then when one account is receiving implies that there is another account which is giving. This aspect represents the cause-effect relationship. The cause-effect relationship is grounded on the second philosophical perspective which has been explored in this paper that is causality. Applying causality perspective in this paper, it is argued that debit entry causally depends on credit entry and vice versa. Last perspective is mathematical modelling in the sense that DEB observe mathematical modelling and mathematical principle of duality grounded on algebra as well as accountophysics which utilizes momentum of particles.

The most notable implication of this paper apart from its contributions which have been already argued is that it will help to have a better conceptualization of DEB. Consequently this will help instructors and practitioners to have a good mastery of which accounts to debit and which accounts to credit consistent with the argument provided by Clevenger (1943) that better accounting skills goes with mastery of identifying which accounts to debit and which accounts to credit when it comes to analysis of the accounting transactions. Likewise, instructor can benefit by teaching DEB which is theoretically grounded hence not getting threatened by the terms debit and credits as revealed by (Ingram, 1998; Pincus, 1997; Vangermeersch, 1997). However, this study has a number of limitations. First, it has not provided practical examples (just mathematical formulation as adopted from Ellerman (2014) for illustration) hence may not be used directly in the undergraduate classroom. This is because this paper is not technical notes to the students. Second, it only applied three theoretical perspectives namely figurative language, causality and mathematical modelling. As such future studies may consider other theories and philosophical assumptions which may be grounded in technological, political and sociological settings. Third, on mathematical assumptions, it only covers limited scope of modelling focus on algebraic models, however, other mathematical approach such geometry, matrix and so on can be applied as well. Lastly, being a perspective paper (as argued by Narula, 2024), it has not analysed prior literature based on the three theoretical perspectives as areas of thematic analysis. As such future studies may extend this study by analysing various literature based on the three theoretical themes as thematic areas. This can be achieved particularly through the use of bibliometric method. While this paper focused on DEB, there is also a need to explore theoretical underpinnings of alternatives to DEB such as Triple Entry Bookkeeping (as proposed by Ijiri and now being advocated under technological development) as well as Resource-Events-Actors (REA) as proposed by McCarthy (1982) and advanced by Dunn and McCarthy (1997).

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Notes

[1] Accounting related literature here is broadly defined to include economics, mathematics, physics and philosophy. These are disciplines which have contributed significantly to the formulation of double entry bookkeeping in short DEB (Aho, 2005; Bardoscia et al., 2021; Demski et.,2006; Fellingham, 2017; Fellingham & Lin, 2020; Fellingham & Schroeder, 2006; Fellingham, Lin & Schroeder, 2022).

[2] Arthur Cayley, born 1821 and died 1895 was a British Mathematician who worked on algebra and postulated Cayley-Hamilton theorem which considers that square matrix is not of its own characteristic polynomial. Cayley was in the same league as Isaac Newton hence it is not surprising that Sombart compared DEB with these scientists' discoveries because they also praised and advocated the richness of DEB.

[3] The connection between accounting conceptualization and economic thinking is well articulated by Karl Polanyi in his book *Socialist Accounting* (Sozialistische Rechnungslegung) of 1922 (Bockman, Fischer & Woodruff, 2016). However, the important point on usefulness of DEB is that it is widely applied in the economic system hence the computation of national income (National Income Accounting) as well as development of monetary policies and fiscal policies are all grounded in DEB perspective (Bindseil, 2004; IMF, 2009; United Nations 2008).

[4] The word "manipulation" here is based on mathematical basis and not financial reporting basis which is more grounded on linguistic perspective (in plain English). While financial reporting standards do not allow manipulation which is taken literary from an English language to mean deceiving or cheating, mathematically manipulation means skills or process of mathematical computation to generate some meaningful values (Carver, 1937; Irwin & Britt, 2005) using mathematical operations which include additions, subtractions, multiplications and divisions.

[5] The issue of accrual accounting requires detailed discussion and expounding in order to show that accrual perspective is more of thinking than just a mere opposite of cash accounting. It is a perspective which deals with scientific evidence in the sense that for accounting transactions to be recognized and treated in accounting, there is a need to accumulate sufficient and appropriate accounting evidences. And these evidences have to be gauged with the objective criteria which in accounting is money (i.e. cash). As such, accrual accounting considers transactions in continuum perspective and those nearing (nearness to cash) are recognised first than those which are not. In other words, accrual accounting is concerned with culmination to cash not the opposite of cash. This is well explained by White (1937) who argued that accrual perspective is a scientific accounting theory which recognises that realization of income is not solely attached to the act of sale or some other specific events in operation but to the entire process of production and that the expenses incurred in the entire process cannot be measured by the cash payments nor the gross revenue by the cash receipts of the period.

[6] Dynamic accounting which is based on dynamic accounting theory as put by Eugen Schmalenbach (1889) (Schmalenbach, 1959) is concerned with flow perspective (action-oriented) hence focused on profit determination making income statement a central issue while static accounting which is based on static accounting theory as developed by Herman Veit Simon (1886) is concerned with stock perspective (status-oriented) hence balance sheet is crucial and profit is merely considered as by-product of accounting process because it is taken as an increase in the net assets during the accounting period (Biondi, 2008; 2012; Eerma, 2014; Hommel & Schmitz, 2013; Mattessich, 2013; Näsi et al, 2013; Richard, 2013; Schmalenbach,1959).

[7] Fabio Besta was born in Teglio di Valtellina (Sondrio, Italy) on 1845 and died on 1922 was accounting thinker and theorist who made great contribution in modern accounting theory such as facilitating the shift from personalistic to non-personalistic (or materialistic) theory of accounts. Professor Fabio Besta who was known throughout Italy as the *Maestro di Ragioneria (Master of Accountancy) or Maestro Insuperato (Unsurpassed Master)* authored the popular book called *La Ragioneria (The Accounting)*. His accounting thoughts were based on value-based theory and equity centred accounting system. As such he developed positive theory of accounts (*Teorica positiva del conto) or famously known as value theory of accounts (teorica dei conti ai valori).* Based on ancient bookkeeping literature he established the fact that the first accounts used in bookkeeping were those opened to real debtors and creditors, in which the terms *dare* (debit) and *avere* (credit) meant respectively "to give" and "to have." *Dare* became associated with debit mutations and *avere* with credit mutations of all assets and liabilities. Hence, *dare* necessarily meant a confirmation of credit mutations and *avere* a confirmation of debit mutations of assets or liabilities (for more details see Galassi & Mattessich, 2004; Kuter et al., 2017; Peragallo, 1938; Sargiacomo, Servalli & Andrei, 2012; 2018).

[8] According to Most (1972, p. 723), "wealth producing sum" or amount invested for the purposes of obtaining profits was operated from all want-satisfaction objectives of the persons involved. In double entry bookkeeping there was only one

objective: the increase of sum of money. Soll (2014, p. xiv) considers DEB to be a method of exacting control and accurately calculating profit, loss and value of assets.

[9] Even though Luca Pacioli is considered to be the Father of Accounting, the DEB itself was not named after him. However, DEB was not invented by Luca Pacioli because DEB is considered to be in operational almost 200 years before the publication of his work. In that perspective attaching him to DEB may be consistent with what Stephen Stigler calls the 'law of misonomy' (i.e. Stigler's Law of Eponymy) which states that no scientific discovery is named after the discoverer. As Stigler (1980) puts it that a discovery may in fact be named after someone who could not be reasonably counted as even one of its discoverers much less the original one. This can be the case with Luca Pacioli. Two reasons can explain his popularity: one he was the first to put together the procedure; and the second reason the document was published at the time of printing evolution era.

[10] Medici Bank was established by Medici Family in Italy during 1397 to 1499. Its Head Office was in Florence in the Republic of Florence in the present day Italy. This bank was opened by Giovanni de Medici which grew to become the Europe's first bank group. While the invention of DEB came before the establishment of Medici Bank, but because this bank needed a more accurate way of keeping books and minimizing errors, it started using DEB and popularized its use. It could be argued that up to date one of the organizations which are configured along DEB perspective is the banking organization.

[11] The Arab numerals which are sometimes called Hindu-Arabic (because it is considered that Arab traders obtained these numerals from India [Dutta, 2023; Ramakrishna Rao, 2002]) were introduced in europe by Italian Merchant and Mathematician called Leonardo da Pisa popularly known as Fibonacci in 1207 (Peragallo, 1938).

[12] The serendipity is unplanned fortunate discovery which occurs throughout the history of scientific innovation and discovery. Serendipity involves with chances in scientific discovery as argued by Beveridge (1957, p.31) … *or at least had an element of chance in them, especially the most important and revolutionary ones. It is scarcely possible to foresee a discovery that breaks really new ground because it is not often in accord with current beliefs.* In case of DEB, the society particularly bankers, merchants, mathematician, philosophers and others tried different techniques until when they arrived at it. Therefore, discovery of DEB followed the same pattern indicating scientific rigour in accounting field hence no surprise that it was celebrated by both mathematicians such as Arthur Cayley, philosophers such as Werner Sombart as well as economists such as Joseph Schumpeter.

[13]Up to this point we have not explained anything about single entry bookkeeping (SEB) because this paper is focused on DEB. However, the difference between the two can be identified clearly through figurative meaning. As argued by Ellerman (2014) that the virtue of DEB as opposed to SEB is not about its mathematical formulation but human properties. For a good discussion on the difference between SEB and DEB see Esquerré (1917) Chapter V to Chapter VI (pp. 54 – 79). Likewise, this paper does not dwell on alternative approaches to DEB such as Triple Entry Bookkeeping, Blockchain as well as Artificial Intelligence (Cai, 2021; Faccia & Mosteanu, 2019; Garanina, Ranta, & Dumay, 2022; Grigg, 2024; Ijiri, 1986; Rahmawati et al., 2023; Schmidt, P& Vejzagić, 2024). While they are considered to be alternative approaches, as far as accounting is concerned they are grounded on DEB basis, they are also considered to be unsatisfactory and not practical, as such the mastery of DEB is still crucial.

[14] This is based on the principle of determination which considers that everything is determined in accordance with laws by something else. This something else may be both the external and the internal conditions of the object in question.

[15] This basically makes deterministic interpretation of causality which considers that A causes B when A must always be followed by B. This is contrary to probabilistic causality in the sense that the occurrence of A increases the likelihood of B's occurrence ($P[B/A] \geq P[B]$) where $P[B/A]$ is the conditional probability that B will occur given the information that A occurred and $P[B]$ is the probability that B will occur having no knowledge whether A did or did not occur. Paraphrasing this in terms of accounting, we consider that Liabilities (L) occurrence increases the likelihood (here assume that liabilities are different from equity) of Asset (A)'s occurrence i.e. ($P[A/L] \geq P[A]$) where $P[A/L]$ is the conditional probability that A (assets) will occur given the information that L (liabilities) occurred.

[16]The manipulationist theory of causality merges counterfactual and manipulation approaches and this is very popular among Philosophers of Economics as admitted by Hausman and Woodward (2004, p. 856) as cited by Maziarz (2020, p. 154): …(p)eople do not expect spontaneous correlations, and they do expect that there will be systematic relationship between (in)dependence relationship when intervention variables are off and they are on, so that they can use each kind of information to learn about others.

[17] A short accounting equation is given by Assets = Liabilities+Equity, however extended one is given by Assets = Liabilities+ Assets=Liabilities+CC+BRE+R−E−D. In this case CC=Contributed Capital (capital provided by the original shareholders),
BRE=Beginning Retained Earnings (earnings notdistributed to stockholders from the previous period), R=Revenue (what is generated from the on-going operation of the company), E=Expenses (costs incurred to run operations of the business), D=Dividends (earnings distributed to the shareholders of the company). Given the extended accounting equation, for example if firm sales, then revenue (R) will increase this will correspond with the increase in Assets (A) in terms of either cash

or accounts receivable hence improving the financial position of the firm. An opposite example is when the firm incur the expenses (E) this will decrease equity on the right side of the equation as well as assets on left side of the equation. But in case the expenses are not settled in cash (say accrued expenses), then assets side will not be affected but will reduce equity side and increase liability. In this second scenario total financial position will not be affected, but the composition of items will be affected.

[18] According to Bunge, semi-causalism is based on eclectic theory. This theory recognizes the validity of causation in certain areas together with unrestricted validity of other categories. This sometimes is called nomic pluralism (Bunge, 2017[1959]).

[19] In nutshell momentum is a vector quantity also known as physical vector which is a product of unit of measurement and vector numerical (unitless) which has both magnitude and direction. Due to its direction it can be used to predict the resulting direction and speed of motion of objects after they collide. Single momentum of particle is the product of two quantities mass and velocity (i.e. p = mv). If there are many particles, p is the vector sum of their momenta and is given as $p = \sum_i m_i v_i$ or p = mv_{cm} if one or more particles are moving. This momentum of particles while has been in the field of mathematics and physics, has also been applied in accounting. Among the first pioneer in the field of accounting is Yuri Ijiri (i.e. Ijiri, 1986) who considered momentum as the rate at which income is being earned and is measured in monetary units per period such as dollars per year or month. Connecting with single momentum of particles, Ijiri (1986, p. 747) argues that ... in mechanics, force is defined as mass (m) times acceleration (a). In the accounting interpretation, force may be defined as investment times acceleration (rate of change in yield), e.g., $\$10/mo^2$=\$10,000 x0.1 $\%/mo^2$, namely the yield, stated as a fraction per month, increases by 0.1 Wo/mo for each one-month duration of, say, an inflationary force. If wealth is W, momentum is M, and force is F, then M=dW/dt, and F=dM/dt =d^2W/dt^2. Here, the measurement of wealth is the primary measurement and the measurement of momentum and that of force are derived measurements. While Ijiri (1986) consider momentum accounting as an extension of DEB, Fischer and Braun (2003), Fischer and Braun (2003a) as well as Braun (2001) they consider momentum of particles as one of the mathematical and physics explanation of DEB. Fischer and Braun (2003a, p. 268): *Bookkeeping … uses an income statement … to measure the profit of an agent over a given time span. Profit increases if assets increase or liabilities decrease, likewise it decreases if assets decrease or liabilities increase. In the mechanical picture, we derive the same result by calculating the momentum change over time for the particles of an agent. Since momentum change over time is the physical definition of a force, the profit of an income statement is derived from the forces which accelerate and decelerate the particles of an agent over a given time span. We therefore see that the income statement is the time derivative of assets and liabilities. Income statement and bookkeeping are the two representations of double entry bookkeeping.*

[20] We introduce the term accountophysics to appreciate the combination of accounting and physics as well as inspiration shown by physicists to address accounting problem consistent with econophysics (see Fischer & Braun, 2003a, 2003b; Melse, 2008). Accountophysics basically indicate high level of conceptualization whereby accounting issues are assessed and investigated using physics-based approaches.

[21]The details here are taken from Ellerman (2014). According to Ellerman (2014), Pacioli group construction has the following properties ;

i. Uses unsigned numbers (no sign positive or negative associated with them i.e. numeric values that only represent nonnegative integers zero and positive values)

ii. Generalizes natural numbers to multidimensional hence unsigned or nonnegative numbers are replaced by vectors of nonnegative numbers (i.e. ordered lists of negative numbers)

iii. Uses ordered pairs of unsigned numbers as objects. These objects are identified with the T-accounts of DEB with the left hand side LHS number d as debit entry and the right hand side RHS number c as credit entry hence T-account: [d//c] = [debit number//credit number].

iv. Algebraic operations in T-accounts resembles that of an ordered pairs called fractions

v. P=P(N) of natural numbers of N = {0, 1, 2, …} consists of the ordered pairs, [x//y] called T-accounts or T-terms of unsigned whole numbers from N.

vi. There is only addition, but there is also additive inverse, for example [x//y] and [y//x] when added give the zero T-account which is [x//y] + [y//x] = [0//0].

vii. Avoids negative and positive numbers (as already said that it uses unsigned numbers) hence, there is a debit isomorphism which associate [x//y] with x – y and credit isomorphism which associate [x//y] with y – x. To translate from T-accounts [x//y] back and forth to the signed integers Z, there is a need to specify debit and credit isomorphism. As such, balance sheet and income statement T-accounts will be labeled as debit balance and credit balance. If a T-account [x//y] is a debit balance, the corresponding number of balance in the account is x – y and if it is credit balance, then the corresponding number is y – x.

Relating Pacioli group to DEB, Ellerman (2014) provides details and examples as summarized below:

- It starts with equation which is additive (in a number theory a function f(m) is called additive if for $(m_1, m_2) = 1$ one has $f(m_1.m_2) = f(m_1) + f(m_2)$.
- It records the changes in the terms of the equation
- It uses T-account equal to the zero T[0//0] termed as zero-account or zero-term
- It encodes zero-account (zero-term) with the equation of unsigned numbers (x = y + z). In this case LHS term x is encoded a debit balance T-account [x//0] and RHS term y is encoded as credit balance T-account [0//y]. LHS and RHS together will add up to the zero-account [0//0] i.e. [x//0] + [0//y] + [0//z] = [0//0]. Hence, the balance sheet equation is encoded as an equation of zero-account. In this case, ledger in DEB is just the listing of the T-accounts of the balance sheet zero-account leaving out plus sign.
- It obtains the ending equation which is considered to be a balance sheet equation and the changes in equation result from transactions affecting the balance sheet and income statement accounts. This can be illustrated using scalars (a scalar is a single number which has got magnitude only) and vectors (a vector is not a single number and has both magnitude and direction).

In case of using scalars

Beginning ledger of T-accounts
Example:

Assets =	Liabilities +	Equity
15000	10000	5000

Assume the production firm has the following transactions (whereby CU = Currency Unit)

1. CU1200 input inventories are used up and changed directly to equity
2. CU1500 of product is produced and added directly to equity
3. CU800 principal payment is made on loan

In this case, each transaction is then encoded as a transactional zero-term and added to the appropriate terms of the equational zero-account. For example, the first transaction subtracts CU1200 from assets and subtracts CU1200 equity. The assets account is encoded as LHS or debit-balance account so subtracting will be encoded as adding the T-account [0//1200] to it. Equity is encoded as a RHS or credit-balance term, subtracting CU1200 would be encoded as adding [1200//0] to it. This kind of treatment applies to all the remaining transactions.

Hence: Initial ledger + Journal = Ending Ledger will give the following details

	Assets	Liabilities	Equity
Original equation zero-account	[15000//0]	[0//10000]	[0//5000]
+ Transaction 1 zero-term	[0//1200]		[1200//0]
+ Transaction 2 zero-term	[1500//0]		[0//1500]
+ Transaction 3 zero-term	[0//800]	[800//0]	
= Ending equation zero-term	[16500//2000]	[800//10000]	[1200//6500]
= (in reduction form)	[14500//0]	[0//9200]	[0//5300]

This gives ending balance sheet equation (assets = liabilities + Equity) i.e. 14500 = 9200+5300. It could be observed that each transaction is encoded as two or more T-accounts that add to the zero-account which is consistent with double entry principle.

In case of using vectors

As in the case of scalar, double-entry vector accounting is a way of recording changes in equation due to changes in transactions and quantities. It uses unsigned components instead of unsigned scalar. Two vector T-accounts [x//y] and [w//z], are equal if their cross sums are equal i.e. [x//y] = [w//z] iff x +z = y + w.

Given x + …+ y = w + …+ z, each LHS vector x is encoded via debit isomorphism as debit-balance T-account $[x^+//x^-]$ and each RHS vector is encoded via the credit isomorphism as a credit-balance T-account$[w^-//w^+]$. Their decomposition follows Jordan decomposition.

Changes in the various accounts in the beginning equation are recorded as transactions and each transaction must be recorded as valid algebraic operations. In this case, listing of the T-accounts in an equational zero-account without plus sign is done in the ledger, while the listing of transactional zero-terms is done in the journal.

Hence: Beginning Ledger + Journal = Ending Ledger

Ellerman (2014, p.496) example is provided here

… consider a simple model in which there are only three types of property: cash, output (widgets), inputs (half-widgets). These goods will be listed in that order in each three dimensional vector. Let the initial asset vector be

(9000, 40, 50), so the firm has $9000 cash, 40 units of widgets in the output inventory, and 50 units of half-widgets in the input inventory. The firm also has $10000 liability represented by the vector (10000, 0, 0) so the equity vector (assets – liabilities) is given by the (net) property vector (-1000, 40, 50). Thus the initial balance sheet vector equation is:

Assets =	Liabilities +	Equity
(9000, 40,50) =	(10000, 0,0)+	(-1000, 40,50)

This gives initial vector T-accounts in ledger as:

Assets =	Liabilities	Equity
[(9000, 40,50)//(0,0,0)]	[(0,0,0)//(10000, 0,0)]	[(1000, 0,0)//(0,40,50)]

Assume the following physical transactions

1. 30 units of the half-widget inputs are used in production
2. 15 units of widgets are produced
3. 20 units of widgets are sold for $100 each
4. $800 principal payment is made on loan

These transactions are then encoded as transaction zero-term and added to the ledger T-accounts. For example, using-up 30 units of half-widgets input is recoded as crediting 30 inputs units to assets and debiting the 30 units to equity. Thus, transactions will be as follows:

Assets =	Liabilities	Equity
[(9000, 40,50)//(0,0,0)]	[(0,0,0)//(10000, 0,0)]	[(1000, 0,0)//(0,40,50)]
1. [(0,0,0)//(0,0,30)]		[(0, 0,30)//(0,0,0)]
2. [(0,15,0)//(0,0,0)]		[(0, 0,0)//(0,15,0)]
3. [(2000,0,0)//(0,20,0)]		[(0, 20,0)//(2000,0,0)]
4. [(0,0,0)//(800,0,0)]	[(800, 0,0)//(0,0,0)]	
[(11000, 55,50)//(800,20,30)]	[(800,0,0)//(10000, 0,0)]	[(1000, 20,30)//(2000,55,50)]
[(10200, 35,20)//(0,0,0)]	[(0,0,0)//(9200, 0,0)]	[(0, 0,0)//(1000,35,20)]

Hence, ending vector balance sheet equation is

Assets =	Liabilities	Equity
(10200, 35,20)	(9200, 0,0)	(1000, 35,20)

The vector can be collapsed to the scalar accounts of value accounting based on given set of prices (valuation coefficients). In this case physical quantities are multiplied and adding up scalar product (dot product). Scalar product of a price vector times a property vector. Assuming prices per unit (cash, output, input) is given as (1, 100,40) respectively, the following values are obtained:

(1,100,40)*(10200,35,20) = 10200+3500+800 = 14500 which is the same figure as that obtained using scalar.

[22] Macro-accounting (also known as macroeconomic accounting, social accounting, economic accounting or national accounting) is a branch of accounting which is concerned with the application of accounting to the analysis of economic activities of the economy as whole, which is aggregate of economic activities. Macro-accounting covers areas such as national balance sheets, national income accounts, input-output system, flow of funds accounts as well as balance of payments (for more details see, Cenar & Cenar, 2021; Du Rietz, 2024; Lande, 2000; Suzuki, 2003a; 2003b; Yu, 1957;1966).