Influence of Teachers' Mathematical Knowledge for Teaching on Students' Academic Achievement in Secondary Mathematics in Tanzania

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Abstract

This survey investigated the influence of teachers' Mathematics knowledge for teaching on students' achievements in ordinary level Mathematics in Mbeya region in Tanzania. Teachers' questionnaires I and II were administered to 27 ordinary level Mathematics teachers. 5224 Form two students from all community schools in Mbeya did the test. Findings indicate that ordinary level Mathematics teachers have low possession of MKT domains. The differences on students' achievements based on teachers' possession of Common Content Knowledge (KCS), Knowledge of Content and Students (KCS), Knowledge of Content and Curriculum (KCC) and Mathematics Knowledge for Teaching (MKT) were verified to be statistically significant at 95% CI. We recommend the increase of teachers' MKT through in-service training focusing mainly on KCS and SCK which are least possessed MKT domains.

Keywords: *common content knowledge, knowledge of content and students, mathematical knowledge for teaching, ordinary level mathematics,*

Introduction

Raising teachers' mathematical knowledge in an attempt to improve the declining students' performance in the national examinations remains a policy priority in Tanzania. Consequently, improving teachers' mathematical knowledge is a focus of the most notable teacher professional development programmes including Teachers Education Assistance in Mathematics and Science (TEAMS), Science Teacher Improvement Project (STIP), Science Education in Secondary School (SESS), Education Quality Improvement through Pedagogy (EQUIP), Collaboration to Support Mathematics Teachers (COSMAT) in Tanzania (Kitta, 2015). This is because evidence suggests that Mathematics teachers in Tanzania lack essential knowledge for teaching (Dachi, 2018; Osaki, 2009; Urio, 2018). Recent research by Mwinuka (2011), HakiElimu (2015), Kitta (2015) and Lema (2019) confirmed the weak knowledge base for teaching among Mathematics teachers.

Although the weaknesses in teachers' mathematical knowledge necessary for effective teaching is widely acknowledged, understanding, defining and assessing such knowledge remains challenging in the Tanzanian context. This is partly because of disagreements regarding what constitute basic knowledge for effective Mathematics teaching among philosophers (Zuzovsky, 2013). Consequently, the effects of teachers' Mathematical Knowledge for Teaching (MKT) on students' achievement remains inadequately established in the context of Tanzania.

Researchers at the University of Michigan developed a test to evaluate teachers' MKT with six domains and found that if a teacher scores well on a test, his/her students' achievement in Mathematics examinations ascends (Orrill, Ok-Kyeong, Peters, Lischka, Jong, Sanchez & Eli, 2015). However, in the Tanzanian context, the question which remains unanswered is: which domains of teachers' MKT are most closely associated with students' achievements in O-level Mathematics? Further, the extent to which teachers possess knowledge of different domains of MKT has not been established. Our study explored the extent to which O-level Mathematics teachers possess MKT and establishes the relationship between the level of MKT a teacher possesses and students' achievement in O-level Mathematics. This study specifically aimed to address the following questions:

i. What is the difference in students' achievement based on teachers' possession of MKT domains?
ii. What is the difference in students' achievement based on teachers' general possession of MKT?

Conceptual framework: Mathematical knowledge for teaching

Mathematical Knowledge for Teaching (MKT) provides a convincing model for clarifying and measuring teachers' knowledge required for teaching Mathematics (Depaepe, Verschaffel & Kelchtermans, 2013). It provides a framework for establishing the relationship between students' learning achievement and teachers' knowledge base. This is because the framework is founded on empirical research focusing on knowledge teachers require in teaching Mathematics effectively. The ensuing section describes the domains of MKT.

Common Content Knowledge – CCK

Common Content Knowledge is the mathematical knowledge teachers require and it is similar to the mathematical knowledge that other professionals use (Ball & Bass, 2009; Martin et al., 2013). It is the knowledge which allows a person to successfully solve mathematical problems beyond the classroom contexts. This includes ability to perform calculations, knowing the definitions of concepts, or making simple representations. Thus CCK refers to mathematical knowledge not confined to teaching. Teachers need CCK to perform various mathematical works assigned to students (Nolan et al., 2015).

Specialized Content Knowledge - SCK

Specialized Content Knowledge refers to mathematical knowledge that is special to the work of teaching, but not required or known in other mathematically demanding professions (Tsafe, 2013). It is a type of professional knowledge that is used to teach the content of a particular branch of knowledge (Olfos et al., 2014). This includes the ability to choose representations of mathematical ideas, recognising patterns in student errors, evaluating whether student responses show an understanding of key sub-concepts and determining if non-standard approaches are valid. Specialized content knowledge consists of an understanding of mathematical concepts derived from experience in teaching. This includes being able to present the same concept in different ways and understanding different methods of deriving answers to problems (Campbell et al., 2014; Nolan et al., 2015).

Knowledge of Content Horizon - KCH

This is a kind of mathematical marginal knowledge needed in teaching and it comprises knowledge of how different topics in the syllabus are related to topics beyond the curriculum (Ball & Bass, 2009). This knowledge is important for the appropriate sequencing of content; it involves understanding of the mathematical horizon and is evident when the teacher demonstrates a broad understanding of how Mathematics they are teaching relates to the Mathematics curriculum their students will face in the future years (Ball & Bass, 2009; Nolan et al., 2015).

Knowledge of Content and Students - KCS

This constitute the ability to identify common mistakes students make and the strategies students use when solving problems (Adedoyin, 2011). Knowledge of Content and Students (KCS) involves knowledge that combines knowing about students and knowing about Mathematics in a way that enables teachers to foresee what students may think and what they will find confusing, interesting and motivating, and to interpret students spoken and written words (Nolan et al., 2015). Ball et al. (2008) asserts that, KCS includes knowledge about common student conceptions and misconceptions, about what Mathematics students find interesting or challenging and about what students are likely to do with specific tasks.

In the case of teaching, Hine (2015) argues that one form of professional vision is a shift from a focus on pedagogy to students thinking. Conceptions of the same idea, methods of solving problems and carrying out procedures vary among students. Teachers should build on students' perceptive notions and methods in designing and implementing instruction. How teachers think about student thinking potentially correlates to the ways in which teachers teach. Students' achievement and understanding are significantly improved when teachers are aware of how students construct knowledge, familiar with methods that students use when they solve problems, and utilize this knowledge when planning and conducting instruction in Mathematics (Beswick & Goos, 2012; Hine, 2015).

Knowledge of Content and Teaching - KCT

Knowledge of content and teaching constitute awareness of the kind of materials or representations that would be best suited to explaining why and how some standard algorithm works (Beswick & Goos, 2012; Campbell et al., 2014). Knowledge of content and teaching (KCT) combines knowledge about teaching and knowledge about Mathematics particularly when a teacher makes instructional choices depending on the purpose. It is likely to be involved in dependent teaching actions, where, for example, a teacher decides which student should be allowed to contribute and who should wait (Nolan et al., 2015).

Knowledge of Content and Curriculum - KCC

Knowledge of Content and Curriculum constitutes awareness of how topics are arranged both within a school year and over time and ways of using curriculum resources, such as textbooks to organize learning trajectory (Beswick & Goos, 2012; Campbell et al., 2014). When teachers are equipped with curriculum materials that guide them, they can help students learn meaningfully (Stylianides & Stylianides, 2014). Further, curriculum materials teachers use mediate mathematical activity taking place in the classroom (Lim & Guerra, 2013).

Research design

This study employed a survey research design with students' test and MKT questionnaires I and II as instruments. Both instruments were reviewed by curriculum experts from the School of Education of the University of Dar es Salaam. MKT questionnaires were pilot tested in a government secondary school in Mbeya City. Student test was reviewed by district mock examinations moderation panel.

All 27 community secondary schools in Mbeya City were involved in the study. These were selected because they had relatively similar characteristics. For example, almost 90% of these schools were established within a span of four years. Further, all the schools followed a common teaching schedule with similar procedures for allocating subjects and grade levels to teachers. A teacher assigned to teach Form One Mathematics would often continue teaching the same class until students completed Form Two. This justified our choice of Form Two for the study. Also, Form One and Form Three annual examinations and Form Two and Form Four mock examinations are centralised in Mbeya City. Examinations are prepared, supervised and marked at district level. By selecting schools with relatively similar characteristics such as handling examinations, school policies, district policies, equipment and supplies for teaching, students' interest in Mathematics and allocation of funds, it was aimed to minimise effects of extraneous variables.

Students' test

Students' test was made up of 23 questions based on the revised Bloom's taxonomy. There were 3 questions measuring "remembering", 5 questions assessing "understanding" and 5 questions reflecting on "applying". Moreover, 5 questions were assessing "analysing skills", 3 questions were measuring "evaluation proficiency" and 2 questions were reflecting on "creating ability". All 5224 Form Two students in community secondary schools sat for the test which was administered as Mbeya City Basic Mathematics Mock Examination on Thursday 13th September 2018. It was planned to administer the mock examination two months before the Form Two National Assessment (FTNA) which was done on Tuesday 13th November 2018 to allow researchers to compare the average GPA on the two examinations and establish reliability.

The average GPA of students' test was 0.36155230 while GPA of Form Two National Assessment (FTNA) was 0.33454607. A Pearson product moment correlation was run to determine the association between student test and FTNA. There was a strong, positive correlation between student test results and FTNA results, which was statistically significant ($r = 0.991$, $n = 27$, $\rho < 0.001$). Hence the test was reliable. Further, a review by the Mbeya City examination panel and curriculum experts ensured that the test was valid.

MKT questionnaires

To measure teachers' possession of each of the six domains of MKT, we adapted MKT test designed by the researchers at Michigan University (Hill et al., 2008). We modified the original MKT test to suit Tanzanian school context and submitted the test to a Mathematics educator at the University of Dar es Salaam department of Mathematics for expert review. Afterwards, we piloted the test with two experienced Mathematics teachers of one of the schools in Mbeya. The distribution of questions in the final MKT questionnaire is presented in Table 1.

Table 1: *Distribution of Questions on the MKT Questionnaires*

S	Domain	Teachers' questionnaire I	Teachers' questionnaire II	Total
1	**CCK**	17	0	17
2	**SCK**	25	3	28
3	**KCH**	15	0	15
4	**KCS**	17	0	17
5	**KCT**	0	35	35
6	**KCC**	0	23	23

Teachers' questionnaire I and II were marked. Afterwards, teachers' scores on each domain were recorded in descending order, listing name of a teacher with the highest score first and lowest score last. The t-test was used to find if there was a significant difference in students' achievement based on teachers' possession of MKT domains. Results of students taught by teachers positioned in the first quadrant were linked with the results of students taught by teachers placed in the fourth quadrant. The overall percentage score per domain was calculated using the formula:

$$\text{Domain percentage score} = \left[\frac{\text{Teachers' total score}}{\text{Domains allocated marks} \times 27}\right] \times 100\%$$

Results and Discussion

Difference in students' achievement based on teachers' possession of MKT domains

Generally, the results show that teachers possessed MKT domains at varying levels. The trend showing teachers' possession of MKT domains is as follows:

KCH 67.65% > CCK 56.43% > KCC 54.27% > KCT 53.33% > SCK 35.98% > KCS 28.54%

Further, the level of MKT domain a teacher possessed unevenly influenced students' achievement with some domains having more effect than others. Overall, students taught by teachers with higher possession of MKT domains have 50.57% more scores than students taught by teachers with low possession of MKT domain. The trend showing the influence of MKT domains on students' achievements is as follows:

KCC 48.81% > CCK 45.25% > KCS 28.68% > KCT 28.17% > SCK 24.92% > KCH 17.52%

Detailed results showing the levels of teachers' possession of MKT domains and the corresponding students' achievement is presented in Table 2 followed by in-depth analysis.

Table 2: *Students' Achievement based on Teachers' Possession of MKT Domains*

Domain	% score	"ρ"	t	Students mean score		Mean difference	% increase in score	Level of significance at 95% CI
				H	L			
CCK	56.43	0.001	4.879	0.43022	0.29620	0.13402	45.25%	Significant
SCK	35.98	0.121	1.668	0.39497	0.31618	0.07879	24.92%	Not
KCH	67.90	0.231	1.274	0.42233	0.35937	0.06296	17.52%	Not
KCS	28.54	0.049	2.193	0.42164	0.32766	0.09398	28.68%	Significant
KCT	53.33	0.177	1.491	0.39520	0.30835	0.08685	28.17%	Not
KCC	55.48	0.002	3.812	0.45552	0.30610	0.14942	48.81%	Significant
MKT	48.98	0.001	4.418	0.43469	0.28870	0.14599	50.57%	Significant

Note:

- ρ = alpha, t = display of independent sample t-test, H = Mean score of students taught by teachers with high score on test items assessing a named MKT domain, L = Mean score of students taught by teachers with low score on test items assessing a named MKT domain, CI = competence interval.

Overall, data in Table 2 shows that students' achievement varied with the level of MKT domains teachers possessed as described more in the subsequent section.

Students' achievements based on teachers' possession of CCK

Data in Table 2 show that CCK is the second most possessed domain of MKT with teachers scoring an average of 56.43% on MKT questionnaire. This suggests that teachers have moderate common content knowledge of Mathematics. Further, data suggest that students taught by teachers with high score on CCK domain had higher achievement compared to those taught by teachers with low scores. When the mean scores on students' test was compared using independent sample t-test, the mean difference was statistically significant at 95% confidence level because 2-tailed reading was $t(12) = 4.879$ and $\rho < 0.001$. Findings further show that CCK is the second most influential MKT domain. It implies that students taught by teachers with higher scores on CCK domain are likely to achieve higher by 45.25% on a Mathematics test.

Although CCK is the second most possessed domain, the score of 56.43% implies that teachers' knowledge of Mathematics content is moderate. Moderate levels of common content of Mathematics among teachers could be due to poor background of teachers in the subject. This could be because more mathematically capable students are not choosing the teaching career (Kitta, 2015; Lema, 2019). Generally, these findings are consistent with those reported by Perry et al. (2006) in Australia and Alabi (2017) in Nigeria.

Students' achievements based on teachers' possession of SCK

Results in Table 2 show that SCK is the fifth most possessed domain of MKT with teachers scoring an average of 35.98%. This suggests that teachers had low specialised content knowledge of Mathematics. Further, although students taught by teachers with high scores on SCK domain had higher achievement compared to those taught by teachers with low scores, when the mean scores were compared using independent sample t-test, the difference was not statistically significant at 95% confidence level because 2-tailed reading was $t\ (12) = 1.668$ and $\rho = 0.121$. Thus, there is no significant difference on students' achievements based on teachers' possession of SCK domain. Findings further show that SCK had fifth lowest influence on students' achievement in Mathematics compared to other domains. An increase in teachers' possession of SCK was likely to improve students' achievement by 24.92%.

The development of SCK appears to be challenging to teachers because the domain is not emphasised in the initial teacher education, it rather depends on teacher's personal creativity (Ball & Bass, 2008; Mwinuka, 2011). In Tanzania, teacher professional development tailored at enhancing teachers' specialised content knowledge of Mathematics seems inevitable. Recent in-service training efforts are commendable.

Students' achievements based on teachers' possession of KCH

It was found that KCH was the most possessed domain of MKT with teachers scoring an average of 67.90%. This suggests that teachers had high knowledge of content horizon. Further, although students taught by teachers with high scores on KCH domain had higher achievement compared to those taught by teachers with low scores, when the mean scores of students were compared using independent sample t-test, the difference was not statistically significant at 95% confidence level because 2-tailed reading was $t\ (10.014) = 1.274$ and $\rho = 0.231$. Thus, there is no significant difference on students' achievement based on teachers' possession of KCH domains. Further, even though KCH was the most possessed domain, its influence on students' achievement in Mathematics test was the lowest. An increase in teacher's possession of KCH domain was likely to raise students' achievements by 17.52%.

Teachers' high scores on KCH domain could be explained by the fact that 74% of sample teachers possessed a degree as their teaching qualification and 26% have have had diploma. This could be the result of government's efforts to encourage teachers to upgrade their teaching credentials, thus more diploma teachers are upgrading to degree. Moreover, the fact that teachers' possession of KCH domain had least influence on students' achievement could be explained by teachers' lack of content knowledge for quality teaching (Makunja, 2016; Nigicser, 2017). Mathematics education in Tanzania largely focuses on procedural literacy and memorisation instead of problem solving (Osaki, 2009). Consequently, increase in teacher's KCH does not translate into quality classroom teaching.

Students' achievement based on teachers' possession of KCS

It was found that KCS was the least possessed domain of MKT with teachers scoring an average of 28.54%. This suggests that teachers had extremely low knowledge of content and students. Further, although students taught by teachers with high scores on KCS domain had higher achievement compared to those taught by teachers with low scores, when the mean scores of students were compared using independent sample t-test, the difference was statistically significant at 95% confidence level because 2-tailed reading was $t\ (12) = 2.193$ and $\rho = 0.049$. Thus, we concluded that there was significant difference on students' achievement based on teachers' possession of KCS domain. Despite that KCS was the least possessed domain of MKT, it was the third most influential domain. Findings suggest that increase in KCS is likely to raise students' achievement by 28.68%. This implies that teachers' possession of KCS can significantly improve students' achievement.

Teachers' low possession of KCS could be due to lack of emphasis on teaching the knowledge of content and students during initial teacher education. Lee (2018) asserts that Mathematics teachers are the victims of their own schooling and training experiences. They rarely identify students' mistakes and problem-solving strategies and use them as a source of classroom discussions. Much of these practices reflect the teaching they experienced (Osaki, 2009; Scott, 2005). Improving teachers' knowledge of content and students is crucial for teachers to understand students' mathematical thought processes. Hine (2015) argues that teachers should shift their focus from pedagogy to students thinking patterns. Conceptions of the same idea, methods and procedures of solving problems vary among students. When teachers understand these variations and utilize such knowledge when planning and conducting instructions, learning achievement significantly increases (Ball et al., 2008; Molina, 2014).

Students' achievements based on teachers' possession of KCT

Results in Table 2 shows that, KCT was the fourth most possessed domain of MKT with teachers scoring an average of 53.33%. This suggests that teachers possessed substantial knowledge of content and teaching. This could be because the development of KCT domain is emphasised in teacher education and professional development programmes (Kitta, 2015; Mwinuka, 2011). Moreover, although students taught by teachers with high scores on KCT domain had high achievement compared to those taught by teachers with low scores, when the mean scores of students were compared using independent sample t-test, the difference was not statistically significant at 95% confidence level because

2-tailed reading was t (7.469) = 1.491 and ρ = 0.177. Thus, there is no significant difference on students' achievement based on teachers' possession of KCT domain. Teachers' possession of KCT moderately influenced students' achievement. An increase in teachers' knowledge of content and teaching was likely to uplift students' achievement by 28.17%. Although teachers' possession of KCT is moderate, efforts are needed to further improve teachers' knowledge of content and teaching. One barrier to achieving higher levels of KCT is the frequent changes in teacher training duration and subsequent debate on whether the curriculum should focus on subject matter or pedagogy or both. While frequent curriculum changes constrain the development of KCT, endless policy debates limit curriculum implementation to meet contemporary demands (Komba & Mwandaji, 2015; Tarmo &Tilya, 2014).

Students' achievement based on teachers' possession of KCC

It was realised that KCC was the third most possessed domain of MKT with teachers scoring an average of 55.48%. This implies that teachers had moderate knowledge of content and curriculum. Further, data suggest that students taught by teachers with high score on KCC domain had higher achievement compared to those taught by teachers with low scores. When the mean scores on students' test was compared using independent sample t-test, the difference was statistically significant at 95% confidence level because 2-tailed reading was t (12) = 3.812 and ρ = 0.002. Thus, O-level students are more likely to achieve higher in Mathematics when taught by teachers with higher knowledge of content and curriculum. This is further substantiated by high influence of KCC domain on students' achievement. Findings indicate that on average, an increase in teachers' possession of KCC domain is likely to improve students' achievement by 48.81%. For teachers to develop higher levels of

KCC, teacher education programmes should emphasise the learning of subject matter prescribed in the school curriculum. Further, curriculum documents such as syllabuses, textbooks and other supporting resources should be available to teachers.

Students’ achievements based on teachers’ possession of general MKT

Results show that teachers moderately possessed general MKT with an average score of 48.98%. Moreover, students taught by teachers with high score on MKT test had higher achievement compared to those taught by teachers with low scores. When we compared the mean score on students’ test using independent sample t-test, the difference was statistically significant at 95% confidence level because 2-tailed reading was $t(12) = 4.418$ and $\rho = 0.001$. Thus, O-level students are likely to achieve higher in Mathematics when taught by teachers with higher mathematical knowledge for teaching. Results further suggest that teachers’ general possession of MKT domain was more influential compared to any other single domain. This is because an increase in teachers’ possession of MKT was likely to improve students’ achievement in Mathematics by 50.57%. Similarly, Hill et al. (2008) found that US students taught by teachers with high scores on MKT had higher achievement in Mathematics tests. Thus, MKT not only constitute the professional knowledge required for teaching Mathematics, but also contributes to teachers’ instructional quality (Depaepe et al., 2013; Hurrel, 2013). In Tanzania, improving teachers’ mathematical knowledge for teaching must be a priority if students’ performance in Mathematics is to be increased.

Conclusion and Recommendations

The study was set out to determine the extent to which O-level Mathematics teachers possess Mathematical Knowledge for Teaching (MKT). Further, it was aimed to establish the relationship between teachers' possession of different domains of MKT and students' achievement in O-level secondary Mathematics. Overall, the study has shown that teachers possess low levels of MKT domains with varying magnitudes following this trend; KCH > CCK >KCC > KCT >SCK >KCS. The second major finding was that, students taught by teachers with higher levels of MKT domains had higher achievement compared to those taught by teachers with low MKT domains. However, the differences in students' achievement were statistically significant for the CCK, KCC, KCS and general possession of MKT only. The differences in students' achievement based on teachers' possession of KCH, KCT and SCK domains were not statistically significant at 95% confidence level.

Collectively, these results suggest that improving teachers' possession of MKT domains can significantly increase students' learning achievement in O-level secondary Mathematics in Tanzania. Further, efforts to improve teachers Mathematical Knowledge for Teaching should focus more on the least possessed domains. Some of the strategies for improving teachers' MKT include organising teachers' professional development programmes focused on Mathematics topics which were either not covered during the secondary and teacher education or are recently introduced in the syllabus. Further, teacher education programmes for Mathematics teachers could be reviewed and the least possessed domains of teacher knowledge should be emphasised. For example, teacher education could focus on helping student teachers connect Mathematics content to real world experience of students. Lastly, Mathematics teacher induction in which novices are attached to experienced teachers for mentoring should be strengthened. Such mentoring should involve episodes of lesson observations involving both the mentor and mentee.

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