# TOOTH MASS REGULATION TECHNIQUE FOR VIBRATION AND NOISE REDUCTION INASYNCHRONOUS ELECTRICAL MACHINES

# Dominic J. Chambega,

University of Dar es salaam, Department of Electrical Engineering, P.O. Box 35131, Dar es Salaam. Tanzania

#### **ABSTRACT**

All industries use energy conversion units a majority of which are asynchronous electrical machines. But energy conversion units emit noise resulting from vibrations and this destroys the environments within which the converters are situated. This paper proposes a technique by which the tooth mass of asynchronous electrical machines can be regulated to reduce vibrations and hence noise emitted by the machines so as to make the environment almost noise free.

#### INTRODUCTION

The reaction of stator and rotor fields due to higher harmonics at skewing the teeth causes vibrations, Fig. 1. It has been shown through experiments and calculations that vibrations caused by 1st slot harmonic at tooth skewing with tooth pitch are sizeably smaller than at straight teeth [1,2]. This is particularly so for those motors in which the ratio of numbers  $z_1$  and  $z_2$  which are the numbers of teeth in the stator and rotor respectively are not compatible. This paper analyses how skewing can be used to reduce the noise level emitted by electrical machines. In addition, it proposes a method by which teeth mass can be regulated to improve vibrations within these machines and hence reduce further the noise level. The testing of the proposed technique is done using an asynchronous machine.

# THE EFFECT OF TOOTH SKEWING

In this analysis the core of the machine is considered as a cylindrical casing exposed to a system with r number of waves which periodically change with time and along whose periphery radial and tangential forces are distributed symmetrically. Figure 2 shows details of the distribution of the waves being considered.

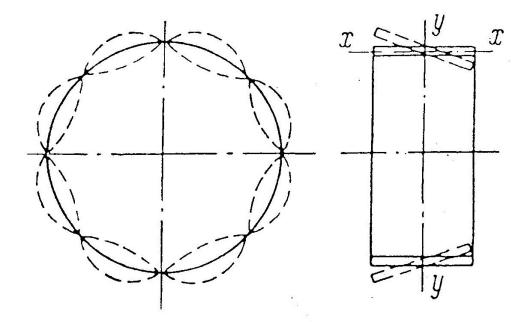
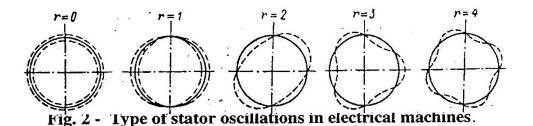


Fig. 1 The form of oscillations of a stator with skewed slots



Magnetic field in the gap at straight teeth in the stator with skewed teeth in the rotor may be described by the following expressions:

(a) Field harmonic of the stator  $b_{v} = B_{v} \cos (v\vartheta - \omega_{1t} - \varphi_{vr})$ ie without change in relation to the usual equation [3]:  $b_{v} = f_{v} \Lambda_{0} = B_{v} \cos (v\vartheta - \omega_{1}t - \varphi_{1})$ (2)

(b) Field harmonic of the rotor [4]:  $b_{tt} = B_{tt} \cos \left[\mu(\vartheta - \gamma_s Z/l_t) - \omega_{tt} + \varphi_{tt}\right]$ (3)

where  $\gamma_s = 2b_s/D_r$  - central angle of tooth skewing  $b_s$  - arc of rotor tooth skewing  $D_r$  - Diameter of the rotor  $l_t$  - length of rotor plates

Different from the usual equation which is of the following form [3,4]:

$$b_{\mu} = f_{\mu} \Lambda_{o} = B_{\mu} \cos (\mu \vartheta - \omega_{\mu} t - \varphi_{2}) \quad (4)$$

equation (3) takes into account the additional phase angle  $\mu\gamma Z/l_t$  due to tooth skewing.

Both of the field waves in accordance with equations (1) and (3) produce radial power waves

$$p_r = P_r \cos (r\theta - \omega_r t - \varphi_r - \mu \gamma z/l_t)$$
 (5)

where 
$$P_r = \left(\frac{B_{\gamma}}{5000}\right) \left(\frac{B_{\mu}}{5000}\right) kgf/cm^2$$
 (6)

$$r = \mu \pm \gamma$$
;  $\omega_r = \pm \omega_1$ ;  $\varphi_r = \varphi_{\gamma r} \pm \varphi_{\mu r}$ 

At studying the effect of tooth skewing on the level of vibrations of the stator it is possible to ideally introduce a variable sign radial force laced at every elementary plate in the form of a rotating vector, Fig. 3, the projection of which on the radius gives a value of an exciting force in time. At straight teeth elementary plates taken along the stator bore will be excited by magnetic forces with same values and phases, Fig. 3 (a).

The resultant force acting on a strip of thickness 1 cm and stator length  $l_t$  is the algebraic sum of all elementary forces. The full cycle of change of these forces takes place within a time through which the rotor moves by one division equal to the length of the wave  $\mu$  and  $\gamma$  depending on where the skewing has been done - in the rotor or stator.

In the presence of tooth skewing phase vector forces will change linearly from one stator end to the other. If  $\gamma_s$  is the central angle of tooth skewing along the stator length, then the opposing forces at the stator end are shifted in phase by  $\mu\gamma_s$  while the phase forces on whichever arbitrary plate situated at a distance z from the middle of the stator equal  $\mu\gamma_s$  z/l<sub>1</sub>.

Figures 3, b-d, show space distribution of forces at tooth skewing of 0.25, 0.5 and 1 length of the rotor wave of the order  $\mu$ . The ends of the force vectors describe a cylindrical spiral which rotates by 900, 1800 and 3600

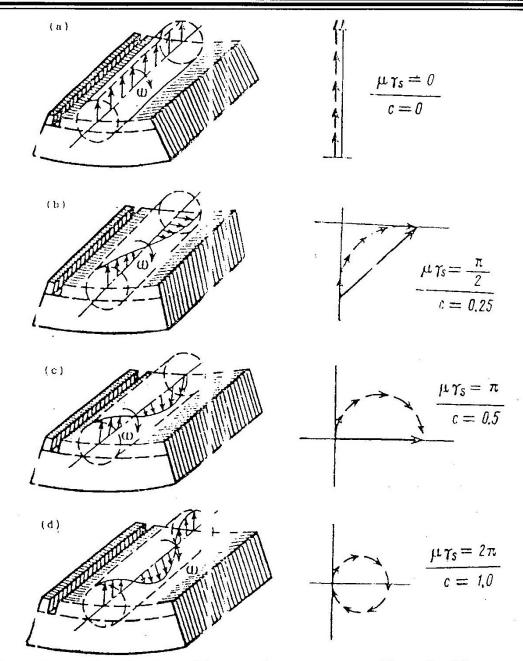


Fig. 3 Space distribution of forces along a stator with and without skewed slots.

along the stator length. The resultant force due to all elementary force vectors confined within one plane equals to the geometrical sum of the forces.

As seen from Fig. 3, as one increases the skewing value the resultant vector drops and remains equal to zero at  $\mu \gamma_s = 2$ . On increasing further the value of tooth skewing what happens is a repetitive cycle of increment

and decrement of radial forces which stands at zero when  $\mu \gamma_s = 4$  and so on and so forth. Stator vibrations, Fig. 1, occur relative to line x - x perpendicular to the machine axis. Taking into account the non-uniform distribution of radial forces along the length of the machine, when calculating deformation of the elbows of the plates of the active steel of the stator the average value of specific force along the length is used, i.e.

$$p_{rt} = \frac{1}{l_r} \int_{-l_r^2}^{l_r^2} p_r dz = P_r \frac{\sin \frac{\mu \gamma_s}{2}}{\frac{\mu \gamma_s}{2}} \cos (r \vartheta - \omega_r t - \varphi_r)$$
(7)

where the expression

$$\frac{\sin \frac{\mu \gamma_s}{2}}{\mu \gamma_s} = k_{s\mu}$$

represents the winding skewing factor for  $\mu$ -th harmonic in the rotor.

Under the circumstances, radial forces acting on the stator with skewed teeth decrease in proportion to the skewing factor. For calculating torsional moment it is necessary to integrate the torque due to elementary forces  $p_r$  dz along the whole length of the plates:

$$m_{\tau} = \int_{-1/2}^{1/2} p z dz \tag{8}$$

The result of the integral in equation (8) is as follows:

$$m_{\tau} = \frac{P I_{t}^{2}}{\mu \gamma_{s}} \left[ \cos \frac{\mu \gamma_{s}}{2} - \frac{\sin \mu \gamma_{s}}{\frac{2}{2}} \right] \sin(r\theta - \omega_{r} t - \varphi_{r})$$
(9)

The amplitude of the torsional moment:

$$M_{\tau} = P_{r} \frac{l_{t}^{2} \frac{\cos \frac{\mu \gamma_{s}}{2} - k_{s\mu}}{\frac{\mu \gamma_{s}}{2}} = P_{r} \frac{l_{t}^{2}}{2} q_{s\mu}$$

The discussion above is also valid in relation to tooth skewing in the stator with the only difference that in the the expressions coefficient  $K_{s\mu}$  and  $q_{s\mu}$  index  $\mu$  has to be substituted by index  $\gamma$ .

## REGULATION OF TOOTH WEIGHT

The effect of tooth skewing in the stator or rotor on the level of vibrations may be calculated using torque equilibrium equations applicable to elements in the yoke of the stator shown as figure 4 in the form of a ring of rectangular cross-sectional area with dimensions of stator at the base. The effect of tooth weight and stator winding may be taken into account by increasing slowly the mass of the base.

Differential equation describing movement of mass of the element is of the following form:

$$J_{p}R_{c}\phi + \frac{EJ_{x}}{R_{c}} \phi - \frac{GJ_{\tau}}{R} \phi'' = M_{\tau}$$
(11)

Here the derivative  $\phi$  is by time;  $\phi$ " is by space coordinates;

$$J_p = \frac{m(l_t^2 + h^2)}{12}$$
 polar moment of inertia of the element mass of the

stator (where m = Q/(2 R cq); Q - mass of the stator  $J_x = h l_t^3/12$  moment of inertia of the cross section of the stator relative x - x axis;  $J_\tau = \eta l_t h^3$  moment of inertia of the cross section of the stator at torsion where  $\eta$  is chosen from the following values [5]:

The shear modules  $G = 0.8.106 \text{ kgf/cm}^2$  (for steel)

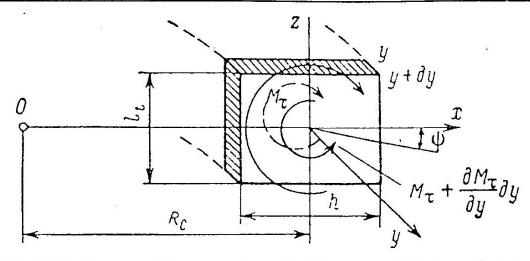


Fig.4 The forces acting on a piece of stator core with skewed slots

In equation 11 torsional moment  $M_{\tau}$  caused by radial forces balances the torques due to forces of inertial  $J_p$   $R_c$   $\phi$  and torques necessitated by forces of elasticity at torsion

$$\frac{GJ_{\tau}}{R_c} \varphi''$$
 and bend  $\frac{EJ_z}{R_c} \varphi$ .

From solutions of homogeneous differential equations it is possible to obtain the frequency of natural torsional oscillations of the stator ( for r = 0, 2, 3, 4 ...)

$$\dot{f}_c = \frac{1}{2\pi} \sqrt{\frac{1}{J_p \lambda_t}} \tag{12}$$

where compliance of the stator at torsion

$$\dot{f}_c = \frac{1}{2\pi} \sqrt{\frac{1}{J_p \lambda_t}}$$

The case when r = 1 is not of interest and hence is not considered.

From the solution of the differential equation 11 for the amplitude of the angular speed of the oscillations the following is obtained:

$$\varphi = \frac{T_{\tau}}{\omega J_{p} - \frac{1}{\omega \lambda_{t}}}$$
(13)

Taking into account that the acoustics power is proportional to the square of the velocity of the oscillations, the effective radial amplitude of the velocity of the oscillations become:

$$y_t = \sqrt{\frac{1}{l_t} \int_{-l/2}^{+l/2} (z\varphi)^2 dz} = \varphi \frac{l_t}{2\sqrt{3}}$$
 (14)

or 
$$y_t = \frac{T_t I_t}{2\sqrt{3}\left(\omega I_p - \frac{1}{\omega \lambda_t}\right)}$$
 (15)

where z<sub>t</sub> - mechanical impedance of the stator during torsion.

The resultant amplitude of the vibrations due to bending and twisting

$$y = \sqrt{y_b^2 + y_t^2} \tag{16}$$

where the vibrations of the bend considering tooth skewing

$$y_b = \frac{p k_{s\mu}}{\omega_m - \frac{1}{\omega \lambda_b}}$$
 (17)

The physical picture discussed above for the cause of vibrations at skewed teeth and the expressions for computations are also applicable in computations of magnetic noise in synchronous and d.c. machines.

## **RESULTS**

An asynchronous motor (6kW) was used for experimentation. First computations were done on the level of magnetic vibrations caused by harmonics (v = +39,  $\mu = -41$ ,  $\omega = 3740$  l/sec, r = 2) with skewed teeth in the rotor at one tooth division. The computations revealed that the use of

skewed teeth reduced the noise level by 14dB to 75dB while the regulation of tooth mass at same angle skewed teeth the magnetic vibrations could further be reduced by 3 dB to 72 dB. The computations were then used for comparison with results obtained experimentally under the same conditions. The results obtained experimentally were similar to those obtained by computation.

# **CONCLUSION**

The paper has proposed a technique by which noise can be reduced by regulating the tooth mass. The technique was tested using a 6 kW asynchronous machine and the results obtained clearly support the use of the technique for noise reduction.

#### REFERENCES

- 1. Summers, E.W., Vibration in 2-pole induction motors related to slip frequency. Trans AIEE, April 1955, 69-72.
- 2. Kovacks, K.P., Tow-pole induction motor vibrations caused by homopolar alternating fluxes. Trans IEE on Power Apparatus and Systems, PAS-96, 4, July August 1977.
- 3. Erskine, J.B., A users view of noise and vibration aspects of a.c. induction motors. Colloquium on the design, application and maintenance of large industrial drives, IEE Pub, Nov. 1978, 52-64.
- Qiong, L., Jianguo, J and Fengshi, Z., Modelling for Studding Pulse Propagation in Turbine Generator Stator Windings. Proceedings of the IASTED International conference on High Technology in the Power Industry. Banff, Canada June 6 - 8, 1996 pp.96 - 101.
- 5. Yang, S.J.: Low Noise Electric Motors, IEE Monographs in Electrical and Electronic Engineering, Oxford Science Pub, 1981.