# OPTIMAL SIZING OF INTEGRATED PROCESS PLANTS

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#### ABSTRACT

The problem of establishing optimum capacity of each plant and material stores in a flowsheet consisting of a number of integrated plants has been formulated as a linear programming (LP) model. In the LP model, process constraints (mass and energy balance), economic constraints (material transport and storage costs) and scheduling are taken into account.

Guidelines for formulating the LP model have been proposed. A case study involving a small scale plant for production of crystalline sugar from sugar cane and an ethanol production plant that uses fermented molasses as raw material have been used to illustrate the LP model.

#### INTRODUCTION

In the context of this paper, an integrated plant is defined as one consisting of a number of process plants whose operation depends on one another with regard to the supply of raw materials and/or utilisation of some of the by-products. To allow plants to operate independently, they should be linked through intermediate material stores.

Two important costs that arise due to the presence of the material store are: the cost of working capital (operating cost) required to hold the product, and the capital cost for installation (or renting) of the store. Also, it need not to be overemphasised that there are costs associated with material transfer from plant to store or vice versa. It follows that the design of an integrated plant involves specification of capacities of the individual plants and intermediate material stores which will minimize the material storage and transport costs.

This design consideration seems to have been overlooked in the litera-

ture[1]. It is the intention of this paper to bridge the gap by first presenting general design guidelines followed by a case study.

## PROPOSED DESIGN APPROACH.

A design approach that would give optimum plant size and material stores is proposed to consist of the following steps:

Step 1. Define the integrated plant.

The pre-condition for the applicability of this method is a situation whereby the system is made of several interdependent plants. In such a system, it is not always economical to build a plant downstream of a process that matches the supply rate of materials by the plant upstream. Instead, by using intermediate stores (building, vessel or even an open yard) it is often possible to install a cheaper, smaller plant down-stream. Hence, at this step we locate a storage facility for each of the by-products of one plant which will become the raw material of another plant downstream.

Step 2. Draw up a long term schedule.

At this step the duration for which each plant will be active is proposed. Several factors derived from the environmental constraints (e.g. availability of labour, length of harvesting season, etc.) may indicate as to when each of the plants should be active. For easier visualisation of co-existing active processes over a number of time intervals, the schedule of activities can be represented in a Gantt chart (Fig 2).

Step 3. Write material constraints for each plant.

This step relates flow of materials entering and leaving the plant over a time period. The governing principle is that the rate of material demand by the plant downstream should not exceed the material supply rate by the plant upstream.

$$\begin{pmatrix}
\text{cumulative material} \\
\text{supplied by upstream} \\
\text{plant}
\end{pmatrix} - \begin{pmatrix}
\text{cumulative material} \\
\text{consumed by the} \\
\text{downstream}
\end{pmatrix} \ge 0 \tag{1}$$

Step 4. Write material storage capacity constraints.

To relate material entering the store over a time period, one condition must be fulfilled. That is, material should not be allowed to overflow in the store. This condition is checked by writing constraint equation of the form;

$$\begin{pmatrix} \text{cumulative material} \\ \text{entering the store} \end{pmatrix} - \begin{pmatrix} \text{cumulative material} \\ \text{leaving the store} \end{pmatrix} \le \text{store capacity}$$
 (2)

#### Special case:

In some integrated plants surplus material which should not be kept in the store is produced by the plant upstream. A smaller material store will be required in this case. A strategy to estimate the store capacity in such a situation will differ from case to case - as will be illustrated later in the case study.

Step 5. Write steady state mass balance.

This step relates the ratio of process material between the inputs and outputs for each plant. The governing equation is the steady state mass balance constraint given by

$$\begin{pmatrix}
\text{material into} \\
\text{the plant}
\end{pmatrix} - (\text{coefficient}) \begin{pmatrix}
\text{material out} \\
\text{of the plant}
\end{pmatrix} = 0$$
(3)

Step 6. Steady state energy balance.

At steady state conditions the flow rates of materials entering or leaving the plant are related to utility flow rates according to the constraint of the form;

$$\begin{pmatrix}
\text{material in} \\
\text{process stream}
\end{pmatrix} - (\text{coefficient}) \begin{pmatrix}
\text{material in} \\
\text{utility stream}
\end{pmatrix} = 0$$
(4)

Step 7. Write an equation for the production targets.

We define production target as cumulative amount of material (product or raw material) processed by one of the plants throughout the campaign. This constraint is of the type;

$$\begin{pmatrix} \text{flow rate} \\ \text{of product} \end{pmatrix} \begin{pmatrix} \text{production} \\ \text{time} \end{pmatrix} = \begin{pmatrix} \text{production} \\ \text{target} \end{pmatrix}$$
(5)

Step 8. Write the objective function.

The objective is to minimise material transport and storage cost;

Min: 
$$fn(\sum transpot costs + \sum storage costs)$$
 (6)

## Transport cost.

The material transport cost component for the objective function can be calculated using a general equation;

$$\begin{pmatrix} \text{transport} \\ \text{cost} \end{pmatrix} = \begin{pmatrix} \text{unit costs of} \\ \text{material transport} \end{pmatrix} \begin{pmatrix} \text{quantity of} \\ \text{material transported} \end{pmatrix}$$
(7)

The unit cost of material transport will depend on the means of transporting the material, e.g. by vehicle, human beings, or pumping.

## Storage Cost.

The cost of storage may be interpreted as referring to the cost of renting the premises. It is therefore independent of the actual amount of material in store at any time, but depends on the length of time for which material is stored. So a constraint linking required storage capacity and cost (equation 8), will be used.

# APPLICATION TO THE CASE STUDY

The case study involves two process plants to be integrated. A small scale sugar production plant has been designed by the Institute of Production Innovation and the Faculty of Engineering both of the University of Dar es Salaam, and is already in operation in the field[2]. However, molasses and bagasse, the by-products of the sugar production process are potential energy sources for the production of ethanol by distillation. It is therefore

possible to integrate the two processes.

Whereas capacity of the sugar plant may be assumed known, the optimum capacity of the distillation plant and size of intermediate material stores (bagasse and molasses stores) need to be worked out.

The steps outlined above are therefore illustrated with this case study as follows:

## Step 1. Define the integrated plant.

As shown in Fig 1, the flowsheet includes bagasse and molasses stores in between the sugar and the distillation plant. Further, since cane harvesting is an activity which can be scheduled, a material store for harvested cane is also included between the farm and the sugar plant.

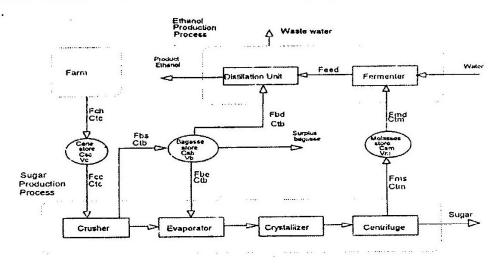


Fig. 1 Material flows in intergrated plants

# Step 2. Draw a long term schedule.

Figure 2 shows an example of scheduling the activities in the integrated plant. Cane harvesting comes first, followed by the sugar production campaign and finally the distillation campaign. For this particular case study typical factors that would influence the schedule are: weather which dictates the duration of the harvesting season (T3-T0), quality of cane deteriorates with storage hence limiting (T1-T0), at the same time the distillation campaign (T5-T2) may continue provided there is bagasse and mo-

lasses in store.

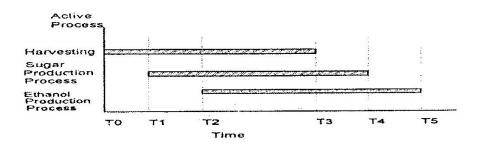


Fig. 2 Schedule of activities in integrated plants (Gantt Chart)

Step 3. Write material balance constraints.

Material balance constraints are written as a function of time for all material flows. To derive the governing constraint equations, reference should be made to Fig 1 and 2 simultaneously. Analogous to the general equation 1, material balance constraint for our case study becomes:

Cane usage;

Bagasse usage;.

$$\begin{pmatrix} bagasse \\ produced \end{pmatrix} - \begin{pmatrix} consumed in \\ sugar proces \end{pmatrix} - \begin{pmatrix} consumed in \\ distillation proces \end{pmatrix} \ge 0$$
(13)

Up to T2 Fbs.(T2 - T1) - Fbe.(T2 - T1) 
$$\geq 0$$
 (14)  
T4 Fbs.(T4 - T1) - Fbe.(T4 - T1) - Fbd. (T4 - T2)  $\geq 0$  (15)  
T5 Fbs.(T4 - T1) - Fbe.(T4 - T1) - Fbd. (T5 - T2)  $\geq 0$  (16)

Molasses usage;

(molasses produced) - (molasses consumed) 
$$\geq 0$$
 (17)  
Up to T4 Fms.(T4 - T1) - Fmd. (T4 - T2)  $\geq 0$  (18)  
T5 Fms.(T4 - T1) - Fmd.(T5 - T2)  $\geq 0$  (19)

Step 4. Write material storage capacity constraints.

As in step 3, Fig 1 and 2 should be referred to simultaneously in deriving material capacity constraints. Similar to equation 2, capacity constraints for each material store are as follows;

Cane store:

T1 Fch.T1 Vc (21)  
T3 Fch.T3 - Fcc.(T3 - T1) 
$$<$$
 Vc (22)

Molasses store;

$$(\text{molasses produced}) - (\text{molasses consumed}) \le Vm$$
 (23)  
 $Fms.(T2 - T1) \le Vm$ 

Up to T2 
$$Fms.(T2 - T1) < Vm$$

(24)

T4 Fms.(T4 - T1) - Fmd.(T4 - T2) 
$$\leq$$
 Vm (25)

Bagasse store;

The general equation for the bagasse store would have been

$$\begin{pmatrix} bagasse \\ produced \end{pmatrix} - \begin{pmatrix} consumed in \\ sugar proces \end{pmatrix} - \begin{pmatrix} consumed in \\ distillation proces \end{pmatrix} \le Vb$$
(26)

However, establishing the capacity of the bagasse store is an example of a special case - a design problem mentioned earlier (step 4). As there would be surplus bagasse which should not be kept in the store (normally surplus bagasse is burnt) the capacity of the store is calculated by considering:

Either demand of bagasse by the sugar plant alone at the beginning of the sugar production campaign before the distillation campaign has started,

i.e. 
$$(T2-T1)$$
. Fbe = Vb (27)

Or demand of bagasse by the distillation plant when the sugar production campaign has ended,

i.e. 
$$(T5 - T4) \cdot Fbd = Vb$$
 (28)

Both equations (equation no. 24 and 25) should be tried in the LP model and the larger value of Vb should be chosen as the required capacity of bagasse store.

Step 5. Write steady state mass balance.

## Sugar plant;

Tthe total amount of cane entering the sugar plant is related to bagasse and molasses produced. Analogous to equation 3 corresponding equations become;

$$Fcc - Kb.Fbs = 0 (29)$$

$$Fcc - Km.Fms = 0 (30)$$

The coefficients Kb and Km and later (step 6 below) Kd and Ke can be established by detailed mass and energy balance considerations round the individual pieces of equipment of the plant.

Step 6. Steady state energy balance.

### Sugar Plant

Flow rate of bagasse as energy source for the sugar process is related to the flow rate of molasses being produced. We therefore have the constraint (refer to equation 4):

$$Fbe - Ke.Fms = 0 (31)$$

## Distillation plant

Similar to equation 5, flow of bagasse is related to molasses raw material according to:

$$Fbd - Kd.Fmd = 0 (32)$$

Step 7. Write an equation for the production targets.

Production capacity of one plant in the integrated plants should be specified. In this case the production target is expressed in terms of cane crushed per year (refer equation 5)

$$Fcc.(T4-T1) = qcane (33)$$

Step 8. Objective function.

#### Transport cost:

The general equation for material transport is given by;

$$\begin{pmatrix} transport \\ cost \end{pmatrix} = \begin{pmatrix} unit cost of \\ material transport \end{pmatrix} \begin{pmatrix} quantity of \\ material transport \end{pmatrix}$$

Hence cost component due to each material being transported is as follow:

Cane transportation;

Bagasse transportation;

Molasses transportation;

#### Storage cost.

The general material cost equation is given by:

Note that daily costs of unit material storage (Csb, Csm, Csc) have units in (Tsh/tonne.day). The duration of material storage has units (days/year) and capacity of store is tonnes. The installation cost (or renting cost) for the store reduced to an annual basis (storage cost per year) is therefore quoted in (Tsh/year). Hence cost equation for each material store becomes;

		· · · · · · · · · · · · · · · · · · ·
Cane storage	Csc.T3.Vc	(41)
Bagasse storage	Csb.(T5 -T1).Vb	(42)
Molasses storage	Csm.(T5 - T1).Vm	(43)

#### RESULTS

The set of equations above may be written in a format to suit a specific LP package. In this work, a VINO software (Schrage, 1986) was used. The advantages of VINO over other LP packages include the spreadsheet capability which allows the sensitivity analysis (what if !) problem to be handled.

Table 1a shows set of values that may serve as input data (independent variables) to the LP model: Economics within the environment would determine the cost data (Ctb..Ctm), whereas the coefficients (Kb..Km) are derived by considering mass and energy balance around the individual plants and the duration of the production campaigns (T1..T5) can be chosen by considering social, economic and environmental factors.

## Table 1. Example of coefficients and results for the LP model

#### Table 1a. Coefficients for LP model

#### Basis;

Cane crushed = 5 tonnes/day (qcane = 525 tonnes/yr) Working days 105 days/yr

#### Basis:

Cane crushed = 5 tonnes/day (qcane = 525 tonnes/yr) Working days 105 days/yr

#### Independent variables

Days	wt/wt	Tsh/tonne	Tsh/tonne
T1 = 1	Kb = 1.66	Ctc = 800	Csc = 50
T2 = 9	Kd = 0.25	Ctb = 235	Csb = 100
T3 = 105	Ke = 1.12	Ctm = 833	Csm = 100
T4 = 106	Km = 20.8		
T5 = 115			

#### Table 1b. Results for LP model

#### Dependent variables

Tonnes/day	Tonnes/day	Tonnes
Fch = 5.0	Fbd = 0.06	Vc = 5.0
Fec = 5.0	Fms = 0.24	Vb = 2.15
Fbs = 3.01	Fmd = 0.24	Vm = 2.14
Fbe == 0.27		

The results of the LP will be a set of values which can yield the following information:

- Fch the rate of cane harvesting in tonnes per day;
- Fcc: the capacity of the sugar plant in terms of tonnes of cane crushed in a day.
- Fbs the corresponding amount of bagasse in tonnes per day required for the sugar production process.
- Fmd the capacity of the distillation plant in terms of tonnes of molasses consumed per day, and
- Fbd the corresponding amount of bagasse fuel in tonnes per day for the distillation process.
- Vc the capacity of cane store, in tonnes of canes,
- Vb the capacity of bagasse store, in tonnes of bagasse and
- Vm the capacity of molasses store, in tonnes of molasses.

# Table 2. LP model with coefficients substituted by numerical values Objective:

```
Min (84000.Fch + 84000.Fcc + 24885.Fbs + 24885.Fbe + 25122.Fbd + 87465.Fms + 88298.Fmd + 5250.Vc + 10600.Vb + 10600.Vm)
```

```
Such that:
```

```
9.Fch - 8.Fcc
105.Fch - 104.Fcc.
105.\text{Fch} - 105.\text{Fcc} = 0
8.Fbs - 8.Fbe 0
105.Fbs - 105.Fbe - 97.Fbd 0
105.Fbs - 105.Fbe - 106.Fbd 0
105.Fms - 97.Fmd 0
105.Fms - 106.Fmd 0
Fch Vc
105.Fch - 104.Fcc Vc
either
        8.Fbe = Vb
        9.Fbd = Vb
* or
8.Fms
        Vm
105.Fms - 97.Fmd. Vm
Fcc - 1.66.Fbs = 0
Fcc - 20.8.Fms = 0
Fbe - 1.12.Fms = 0
```

```
Fbd - 0.25.Fmd = 0

105.Fcc = 525

Fch > 0

Fcc > 0

Fbs > 0

Fbe > 0

Fbd > 0

Fms > 0

Fmd > 0

Vc > 0

Vb > 0

Vm > 0
```

The objective value of the LP model is the annualised cost due to the cost of material storage, and transport costs of materials between plants.

Table 1(b) shows the results of the LP model. A complete listing of the LP model with the coefficients substituted by numerical values is as shown in Table 2. whereas Table 3 is a corresponding VINO spreadsheet.

The LP model developed as above serves as a useful tool in studying the effect of change in the objective value and/or dependent variable due to changes in the independent variables one at a time. In this case specifying different time duration of the production campaigns would result into different capacity of the plant and intermediate stores as follows:

## Effect of reducing length of sugar production campaign

Table 4 and figures 3 to 5 are examples of simulation results. These results show that by reducing the length of sugar production campaign (T4-T1), a larger sugar plant (Fcc) and a larger cane store (Vc) will be required, consequently increasing the cost of the plant (objective value). Sugar production campaign should therefore start as soon as cane harvesting season has started.

<sup>\*</sup> This equation gives the largest value of Vb.

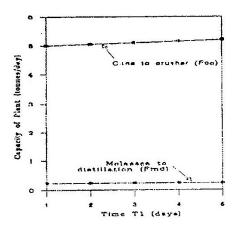
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				Ke 1.12 qeane \$25	0.25 0.75 100	1.66 1.66	Car. 30	T.4 106 833	C: 6 0	90.0 0.0 13	71 70.83
									pecify)	Independent variable (specify)	Independe

Table 3. An LP model for sugar and ethanol production plant (in VINO format)

Table 4 Effect of reducing length of sugar production campaign(T4-T1)

*(T4-T1) days	105	104	103	102	101
T2 tonne/day	9	10	11	12	13
Fcc tonne/day	5.0	5.05	5.10	5.15	5.20
Fmd tonnes/day	0.24	0.24	0.24	0,24	0.25
Vc tonne	5.0	10.0	15.0	20.0 .	25
Vb tonne	2.15	2.17	2.19	2.21	2.24
Vm tonne	2.14	2.16	2.18	2.20	2.22
Obj. Tsh/yr (10 <sup>6</sup> )	1.04	1.06	1.09	1.12	1.15

\* T4 = 106 days



25.0

25.0

26.0

Cane store (Ve)

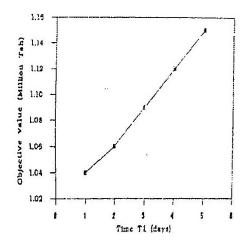
Signature (Ve)

Molasses
etore (Vm)

Time Ti (days)

Fig. 3 Plant capacity vs time T1

Fig. 4 Store capacity vs time T1



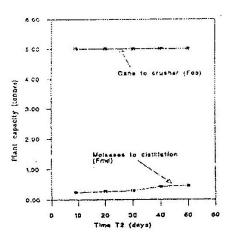


Fig. 5 Objective value vs time T1

Fig. 6 Plant capacity vs time T2

## Effect of reducing length of distillation campaign

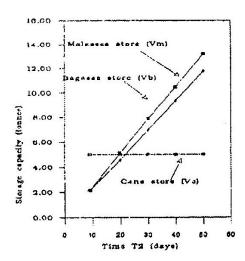
Likewise, Table 5 and Fig 6 to 8 are examples of simulation results and they show that by reducing the length of the distillation campaign (T5-

T2), a larger capacity distillation plant (Fmd) and larger molasses and bagasse stores (Vm and Vb respectively) will be required. Correspondingly the objective value to be minimised increases. Hence distillation campaign should start as soon as the sugar plant has started operating.

Table 5 Effect of reducing length of distillation campaign (T5-T2)

T2 days	9	20	30	40	50
*(T5-T2) days	106	95	85	75	65
Fcc tonne/day	5.0	5.0	5.0	5.0	5.0
Fmd tonnes/day	0.24	0.27	0,3	0.43	0.39
Vc tonne	5.0	5,0	5.0	5.0	5.0
Vb. tonne	2.15	25.11	7.88	10.48*	13.17
Vm tonne	2.14	4.56	6.96	9,36	11.76
Obj. Tsh/yr (10 <sup>6</sup> )	1.04	1.09	1.14	1.2	1.25

<sup>\*</sup> T5 = 115 days



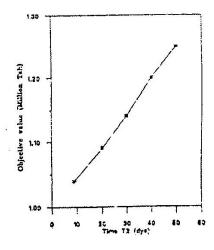


Fig. 7 Store capacity vs time T2 Fig. 8 Objective value vs time T2

# Effect of extending length of the distillation campaign

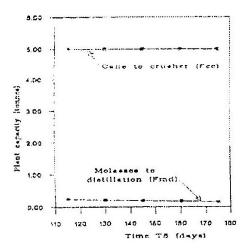
As shown in Table 6 and Fig 9 to 11, by extending the distillation campaign (T5-T2) a smaller capacity distillation plant (Fmd) and molasses store (Vm) will be required. However the objective value tends to increase. This suggests that the distillation campaign should finish just after the

sugar production campaign finishes.

Table 6 Effect of extending length of the distillation campaign (T5-T2

T2 days	9	20	30	40	50
*(T5-T2) days	106	95	85	75	65
Fcc tonne/day	5.0	5.0	5.0	5.0	5.0
Fmd tonnes/day	0.24	0.27	0.3	0.43	0.39
Vc tonne	5.0	5.0	5.0	5.0	5.0
Vb tonne	2.15	25.11	7.88	10.48*	13.17
Vm tonne	2.14	4,56	6.96	9.36	11.76
Obj. Tsh/yr (10 <sup>6</sup> )	1.04	1.09	1.14	1.2	1.25

<sup>\*</sup> T5 = 115 days



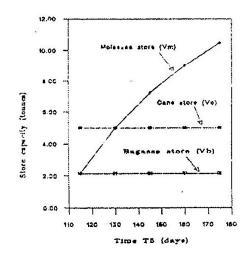


Fig. 9 Plant capacity vs time T7

Fig. 10 Store capacity vs time T5

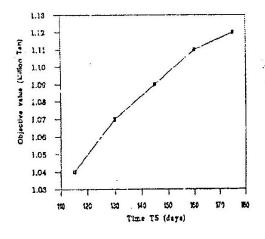


Fig. 11 Objective value vs time T5

#### DISCUSSION

As pointed above the aim of this study was to develop procedure for sizing integrated plants. For that matter a case study involving sugar and alcohol production plants has been used to develop the procedure. In illustrating the procedure results which make sence have been obtained suggesting that the method as proposed in this paper answers the problem it intends to address.

Although the case study involves few process plants (two in this case) it is adequate to assume that the underlying principles can be applied to other cases involving large number of integrated plants.

The case study specifically refers to storage as being the building, and vehicle or human labour as the means of transport. Other forms of storage and transport may be considered for other systems. For example one can model batch and semi-continuous units of a given process plant as an integrated system and accordingly employ design approach as presented in this paper. In such a case storage is achieved through storage vessels whereas transport may be achieved by pumping the process fluids involved from one processing stage/storage vessel to another stage down-stream the process.

# CONCLUSION

A method which involves development of an LP model for establishing optimum capacities of individual plants in an integrated plant has been developed. In addition to the mass and energy balance considerations, the method allows taking into account material transport costs, material storage costs and scheduling.

To illustrate the method, optimum capacity of the distillation plant, and the capacity of bagasse and molasses stores to match with the sugar plant have been worked out. Also, analysis has been carried out which illustrates quite clearly how scheduling of the production campaign affects the capacities of the individual plants and the material stores. For the particular set of data the most economic plant capacities corresponds to all the three campaigns, i.e. cane harvesting, sugar production and distillation plant operation overlapping as much as possible.

# **NOMENCLATURE**

Csb Csc Csm Ctb Ctc Ctm Fbd Fbe	unit cost for intallation of bagasse store, TSh/tonne unit cost for installation of cane store, TSh/tonne unit cost for installation of molasses store, TSh/tonne. unit cost for transporting bagasse, TSh/tonne. unit cost for transporting cane, TSh/tonne. unit cost for transporting molasses, TSh/tonne. unit cost for transporting molasses, TSh/tonne. bagasse consumption rate by distillation process, tonnes/day. bagasse consumption rate by sugar production process, tonnes/
	day.
Fbs	bagasse production rate by sugar plant, tonnes/day.
Fcc	size of sugar plant (cane crushing rate), tonnes/day.
Fch	flow rate of cane harvested, tonnes/day.
Fmd	size of distillation plant (rate of molasses consumption), tonnes/
	day.
Fms	molasses production rate by sugar productio, tonnes/day
	process.
qcane	plant capacity in terms of cane crushed per year, tonnes/day.
	coefficient in mass balance constraint, wt/wt
	coefficient in energy constraint (distillation plant), wt/wt
Vc	capacity of store for cane, tonnes
Vb	capacity of bagasse store, tonnes.
Vm	capacity of molasses store, tonnes.
Tsh	Tanzanian Shilling (US $$1.00 = 700$ Tsh approx. as of the time
of	this research)
T0T5	n.b. Time difference e.g. (T3-T0), (T4-T1) etc. refers to duration of the production campaign and has units in days/year.

# **ACKNOWLEDGEMENT**

The author wishes to acknowledge the sponsorship of DAAD during execution of this study at the University of Leeds. The valuable comments by Dr John Flower of the same university is also appreciated.

#### Kaunde

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The manuscript was received on 1st March 1994 and accepted for publication on 5th May 1995.