Uhandisi Journal Vol. 21 No. 3, December 1997 pp. 69-88

MODELLING RAPID TRANSIENTS USING MACCORMACK METHOD

C. Z. M. Kimambo
Institute of Production Innovation, University of Dar es Salaam
P. O. Box 35075, Dar es Salaam, Tanzania

ABSTRACT

Linebreak problems are a typical example of transient and unsteady flow of fluids in pipeline. The three equations of conservation which mathematically model such flows are well known. Many numerical methods are available and have been used for the solution of these equations. In this paper, the most commonly used numerical methods and their applications are described together with their comparative advantages and drawbacks in various fluid transient phenomena. The methods are the method of characteristics, finite-difference methods, finite-element methods, flux-difference splitting schemes, the method of lines and the wave-plan method. A computer model based on the gamma delta method has been developed for analysis of unsteady and transient flow of natural gas following linebreak in high pressure pipeline. The second-order two-step method of MacCormack was used for numerical solution of the equations of conservation. The procedure and equations used are presented. Validation of the computer model with some experimental data gave encouraging results. The method could be applied for solution of linebreak problems. These results are presented and discussed in this paper. Also the merits of the MacCormack method over the method of characteristics which is the most common method of solution for such problems are discussed.

INTRODUCTION

In the process of developing a computer model for analysis of unsteady flow of fluids in pipelines, one of the critical factors is selection of the numerical method to be used for solution of the basic equations. The various numerical methods for solution of the basic partial differential equations of unsteady fluid flow in a pipeline and a discussion of their applications and suitability for the solution of the unsteady flow equations for a ruptured high-pressure gas pipeline are presented in this section. The shock-capturing explicit finite-difference methods of solution are preferred to the shock fitting scheme using the method of characteristics [1]. Presentations of the basic partial differential equations for the more popular numerical methods were made [2, 3]. Explicit finite-difference schemes range from the single-step first-order schemes to four-step fourth-order schemes. Explicit finite-difference methods integrate the basic partial differential equations by considering the changes in the dependent variables along the directions of the independent variables. This produces the solution values at evenly spaced points in the physical plane. This study focuses on the two-step second-order MacCormack method. Referring to Fig. 1, this method allows explicit calculation of approximate values $A_{(i,t+\Delta t)}$ of the solution at certain node points (i, t+Δt) of a rectangular grid from known exact or approximate values $A_{(i\ ,t)}$ of the solution at another node point (i, t), preferably belonging to the past.

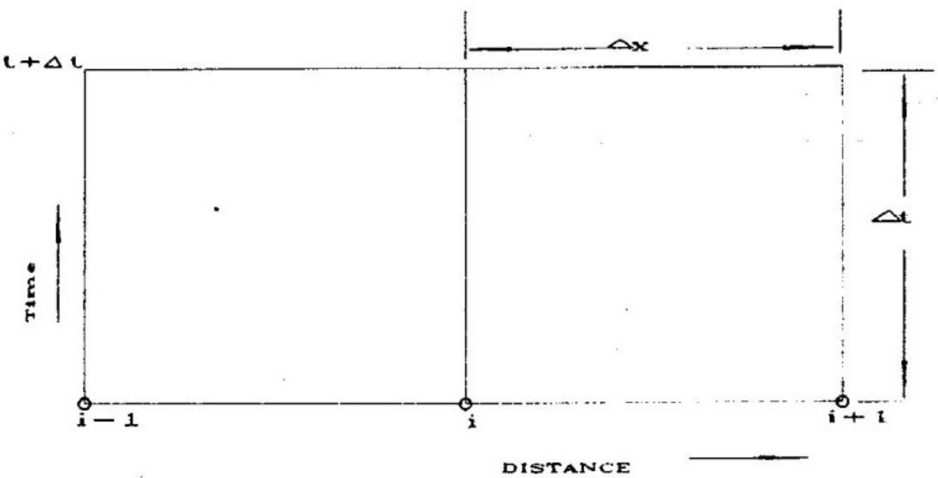

Fig. 1 Calculation Mesh for the Second-order MacCormack Method

The basis of finite-difference formulations is that the differential terms of the dependent variables appearing in the partial differential equations, are expressed in approximate expressions so that a digital computer which performs only standard arithmetic and logical operations can be employed to obtain a solution. Two methods which are used for approximating the differential terms are the Taylor series expansion and polynomials. The approximations of the derivatives may be expressed as either forward, backward or central differences; first-, second-order accurate and so on. The finite-difference approximations are used to replace the derivatives that appear in the partial differential equations. Finite-difference formulation of the partial differential equations can be done in two ways, namely, explicit and implicit formulations. Finite-difference methods are therefore categorised in the two types mentioned above. Obviously, the solution of explicit equations is simpler than the implicit equations. However, implicit formulations are more stable than explicit formulations. In case of rapid transients, where small time-steps and large number of sections are required, the method looses the advantage of fast computation. Implicit finite-difference methods are suitable for the analysis of slow transients on relatively large networks. Among the other categories of numerical methods, there are some which are based on finite-difference formulation. These include the method of characteristics, the flux-difference splitting schemes, the method of lines and the wave-plan method.

Many different explicit finite-difference methods, ranging from single-step first-order accurate to four-step fourth-order accurate schemes, have been developed for the fluid transient equations. Some of the popular methods are: Forward Euler Method, Method of Lax, Lax-Wendroff Single-Step Method, Lax-Wendroff Two-Step Method, Alternating Gradient Method, MacCormack Method, Rusanov-Burstein-Mirin Method, Abarbanel-Gottlieb-Turkel Method, Hopscotch Method, Leap Frog Method, Pseudoviscosity Method and Warming-Kutler-Lomax Third-Order Method. Similarly, many implicit finite-difference methods have been developed and used for the solution of various engineering problems. Those which have been popular for the solution of the partial differential equations describing unsteady fluid flow in pipelines include the following: Fully Implicit Method, Crank-Nicolson Method, Centred Difference Method, Characteristic Finite-Difference Method, Explicit-Implicit Methods, Guy Method, Gear Method, Backward Euler Method, and Beam-Warming Method.

The major advantage of explicit finite-difference methods especially in comparison with the method of characteristics is that they are very simple to programme. The conservative form of the hyperbolic equations for unsteady pipe flow has the favourable property that conservative finite-difference methods applied to it produce solutions which greatly facilitates accurate shock calculation. No special care needs to be taken of the location of shocks, therefore it is suitable for systems in which shocks form. There are no eigenvectors to be computed. Eigenvectors are needed solely for testing stability conditions. There are no linear or non-linear equations to be solved. Explicit finite-difference methods need comparatively little computer memory space since they solve the equations directly rather than simultaneously. Second-order of accuracy is normally regarded as sufficient for the analysis of gas transients. Finite-difference methods produce solution values at evenly spaced points in the physical plane. One of the major disadvantages of finite-difference methods, other than the method of characteristics, is that continuous initial data may propagate along the characteristics thus making it difficult to handle. Explicit methods suffer from stability problems since they are only conditionally stable. Time steps are restricted by a stability criterion, which result in a large amount of computer time being required. They are therefore not suitable for analysis of large systems or unsteady flows over long periods of time. In the presence of shocks, methods of higher than first-order produce considerable overshoot and oscillatory systems. A smoothing parameter for overshoot can tend to smooth out the transient peaks. Unlike the method of characteristics, finite-difference methods are unable to solve the boundary conditions naturally.

The method of characteristics has been described in detail [1]. One of the major drawbacks of the method of characteristics is that if the dependent variables are required at fixed time intervals, a two-dimensional interpolation in the characteristic net is required and this may be quite complicated. However, this drawback has been overcome by the mesh method of characteristics called the method of specified time intervals, which solves the characteristic equations on values for the dependent variables at specified time-distance coordinates. The method of characteristics has many advantages compared with the other numerical methods of solution. In the method of characteristics solution, discontinuities in the initial value may propagate along the characteristics, making it easy to handle them. Large time steps are possible in the natural method, since they are not restricted by a stability criterion. The boundary conditions are also properly posed. It is time consuming to programme on a computer. Discontinuous initial data and shock waves do not lead to solution with overshoot and details are not smeared, in the natural method. Exact solution is possible in the constant coefficient case with two dependent variables regardless of eventual discontinuities in the initial data, in the case of the natural method. No attention needs to be paid to the position of the shocks, in the hybrid method and in general it causes only a small overshoot.

The theoretical basis of the finite-element methods has been covered extensively [4]. Finite-element methods have not been used for gas transients as widely as the finite-difference based methods and thus there are not many of these methods known. Until recent years, only two methods namely the Galerkin Method [5], and more recently [6, 7] and the moving finite-element method, have been used. A weighted residual finite-element method, which uses the Galerkin finite-element method to discretize the equation, was developed [8]. The most popular finite-element methods for fluid transients are therefore the following: Galerkin Method, Spectral Galerkin Method, Spectral Collocation Method, Moving Finite-Element Method, and the Method of Bisgaard-Sørensen-Spangenberg [8]. Finite-element methods have some advantages over finite-difference methods. The former methods can be used to solve virtually any engineering problem for which a differential equation can be written. They have a higher accuracy because cubic hermite splines which should give smallest errors could be used. The major disadvantage of finite-element methods is that they are somewhat complex, with complexity being proportional to the complexity of the differential equations for that particular problem.

The concept of upwinding has been used in local discrete approximations in finite-difference and finite-element methods for many years. Upwind schemes have become popular since the beginning of the 1980's [9, 10, 11], and these methods now play an important role in computational fluid dynamics. They offer a sound theoretical basis of the characteristic theory for hyperbolic systems and are capable of capturing discontinuities. Some of the most common methods for the solution of the basic equations for transient gas flow are the formulation, flux-vector splitting methods and flux-difference splitting methods. Theoretical descriptions of the method of lines has been published [12, 13, 14]. The method of lines is empirical and extremely simple. Higher-order methods can be used for the integration of time, for example fourth-order Runge-Kutta or multi-step predictor-corrector method, which is approximate for parabolic problems. The main advantage of the method of lines is that it offers the possibility of utilizing highly developed software for ordinary differential equations. The method is convenient in particular where lumped-parameter systems of ordinary differential equations in time is required. With the method of lines it is difficult to treat the boundary conditions properly. Application of the wave plan method has mainly been on liquid systems [15, 16], and therefore the basic partial differential equations have been derived from the equations of continuity and momentum. The wave-plan method is more easily applied to complex unsteady flow systems. The wave-plan method is advantageous in making certain types of dynamic response calculations. For example, the response of fluid filled lines having types of axial cross-sectional area distribution for which there would be little hope of obtaining closed-form analytical solutions, can be easily handled. The method readily solves problems in which there is interaction between the structural motion of the conduits and the perturbations in the fluid flowing within the conduit. The main limitation of the wave-plan method when dealing with pipe networks is that the time interval must be chosen small enough to account for pressure waves traversing the shortest pipe section in the network thus requiring frequent calculations.

There has been many studies on fluid transients using the numerical methods described in this section. These have been discussed in detail [17]. Most of the studies are based on gas systems, although a few based on other fluid systems have also been included. The studies are generally grouped into three categories. The first category is that of studies consisting of theoretical reviews and comparisons of various numerical methods of solution of fluid transient problems. The second category consists of practical studies on various fluid transient phenomena, other than linebreak. The third category is that of studies dealing with linebreak (pipe rupture and blowdown) modelling. Based on this analysis it was concluded that explicit finite-difference methods are the most suitable for solution of the basic equations for analysis of linebreak problems. Based on the literature study conducted, the second order-method developed by MacCormack [18] and the third-order method developed by Warming, Kutler and Lomax [19] were the most preferable. The method of characteristic was selected for solution at the boundary points. However, comparative studies on the three numerical methods mentioned above proved that the method of characteristics is the best of the three for modelling rapid transients in ruptured high pressure gas pipelines. The second-order MacCormack method gave satisfactory results.

THE BASIC EQUATIONS OF FLOW

The basic equations for unsteady quasi-one-dimensional flow of real gases through non-rigid and variable cross-sectional area pipes are considered, using the Gamma Delta method, which were derived [20]. The basic equations for unsteady flow equations were further simplified to the following equations:

Continuity equation

$$\frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial x} + \rho \frac{\partial u}{\partial x} = 0 \tag{1}$$

Momentum equation

$$\frac{\partial u}{\partial t} + \frac{1}{\rho} \frac{\partial p}{\partial x} + u \frac{\partial u}{\partial x} = - \frac{\omega}{\rho A} - g \sin\theta \tag{2}$$

Energy equation

$$\frac{\partial p}{\partial t} + u \frac{\partial p}{\partial x} + a^2 \rho \frac{\partial u}{\partial x} = \frac{1}{A} (\delta_S - 1)(\Omega + \omega u) \tag{3}$$

The QUANT software for thermodynamic and transport properties of fluids [21] was used.

In order to write the equations for the second-order MacCormack method, the basic partial differential equations of conservation (equations (1), (2) and (3)) have to be expressed in the following form:

$$\frac{\partial (A)}{\partial t} + \frac{\partial (B)}{\partial x} = C \tag{4}$$

MacCormack [18] used his method for the time dependent Navier-Stokes equations in two dimensions. The equations are linear i.e. body forces and heat transfer were neglected. However, the method can be used in situations where the basic equations are not linear, for example [22] for analysis of transient flow of a compressible single-phase liquid in an elastic pipe. The basic equations in that case were quasi-linear.

The three basic partial differential equations (1), (2) and (3) can be expressed in the matrix form of equation (4), where the matrices A, B and C are as follows:

$$\underline{A} = \begin{bmatrix} p \\ u \\ \rho \end{bmatrix} \tag{5}$$

$$\underline{B} = \begin{bmatrix} u & \rho a^2 & 0 \\ \frac{1}{\rho} & u & 0 \\ 0 & \rho & u \end{bmatrix} \tag{6}$$

$$\underline{C} = \begin{bmatrix} (1 - \delta_s)\frac{(\Omega + \omega u)}{A} \\ \frac{\omega}{\rho A} + g \sin\theta \\ 0 \end{bmatrix} \tag{7}$$

Before performing transient analysis, steady state analysis was performed in order to establish the initial conditions (conditions at t = 0 s) in the pipeline. This was followed by transient analysis before introducing the break boundary conditions.

SOLUTION OF THE BASIC EQUATIONS BY MACCORMACK METHOD

The MacCormack Method is superior to the method of characteristics when Courant number (C_n) differs appreciably from unity. It is inherently dissipative i.e. because it is second-order accurate in both space and time, no special shock capturing approach is needed. It is unconditionally stable if C_n is less than unity and it produces minimal precision loss when C_n moves away from unity. The method permits the use of a grid spacing that is not overly fine even in highly complex cases. As the method is quite efficient, overall computation effort remains reasonable. The MacCormack method could be very well suited for applications of increasing complexity such as two-phase, gas-liquid flow problems and multi-dimensional flow. It is simple and has low development cost. The complete equations can be used without making any simplification.

The finite difference form of equation (4) using the MacCormack method can be written in either of the two alternatives 1 or 2 described below. The subscripts and superscripts used refer to Fig. 1.

ALTERNATIVE 1:

Predictor step (forward difference):

$$A_i^P = A_i^t - B_i^t \frac{\Delta t}{\Delta x}(A_{i+1}^t - A_i^t) - C_i^t \Delta t \tag{8}$$

Corrector step (backward difference):

$$A_i^C = A_i^P - B_i^P \frac{\Delta t}{\Delta x}(A_i^P - A_{i-1}^P) - C_i^P \Delta t \tag{9}$$

ALTERNATIVE 2:

Predictor step (backward difference):

$$A_i^P = A_i^t - B_i^t \frac{\Delta t}{\Delta x}(A_i^t - A_{i-1}^t) - C_i^t \Delta t \tag{10}$$

Corrector step (forward difference):

$$A_i^C = A_i^P - B_i^P \frac{\Delta t}{\Delta x}(A_{i+1}^P - A_i^P) - C_i^P \Delta t \tag{11}$$

Writing the finite difference equations for equations (1), (2) and (3), the following equations were obtained:

ALTERNATIVE 1:

Predictor step (forward difference):

$$\rho_i^{P1} = \rho_i^t - u_i^t \frac{\Delta t}{\Delta x}(\rho_{i+1}^t - \rho_i^t) - \rho_i^t \frac{\Delta t}{\Delta x}(u_{i+1}^t - u_i^t) \tag{12}$$

$$u_i^{P1} = u_i^t - \frac{1}{\rho_i^t}\frac{\Delta t}{\Delta x}\left(p_{i+1}^t - p_i^t\right) - u_i^t\frac{\Delta t}{\Delta x}\left(u_{i+1}^t - u_i^t\right) + \Delta t\left(\frac{\omega_i^t}{\rho_i^t A} + g\sin\theta\right) \tag{13}$$

$$p_i^{P1} = p_i^t - u_i^t\frac{\Delta t}{\Delta x}\left(p_{i+1}^t - p_i^t\right) - [a_i^t]^2\rho_i^t\frac{\Delta t}{\Delta x}\left(u_{i+1}^t - u_i^t\right) + \Delta t\left[\left(1-[\delta_S]_i^t\right)\left(\frac{\Omega_i^t + \omega_i^t u_i^t}{A}\right)\right] \tag{14}$$

Corrector step (backward difference):

$$\rho_i^{C1} = \rho_i^{P1} - u_i^{P1}\frac{\Delta t}{\Delta x}\left(\rho_i^t - \rho_{i-1}^t\right) - \rho_i^t\frac{\Delta t}{\Delta x}\left(u_i^{P1} - u_{i-1}^{P1}\right) \tag{15}$$

$$u_i^{C1} = u_i^{P1} - \frac{1}{\rho_i^{P1}}\frac{\Delta t}{\Delta x}\left(p_i^{P1} - p_{i-1}^{P1}\right) - u_i^{P1}\frac{\Delta t}{\Delta x}\left(u_i^{P1} - u_{i-1}^{P1}\right) + \Delta t\left(\frac{\omega_i^{P1}}{\rho_i^{P1}A} + g\sin\theta\right) \tag{16}$$

$$p_i^{C1} = p_i^{P1} - u_i^{P1}\frac{\Delta t}{\Delta x}\left(p_i^t - p_{i-1}^{P1}\right) - [a_i^{P1}]^2\rho_i^{P1}\frac{\Delta t}{\Delta x}\left(u_i^{P1} - u_{i-1}^{P1}\right) + \Delta t\left[\left(1-[\delta s]_i^{P1}\right)\left(\frac{\Omega_i^{P1} + \omega_i^{P1}u_i^{P1}}{A}\right)\right] \tag{17}$$

ALTERNATIVE 2:

Predictor step (backward difference):

$$\rho_i^{P2} = \rho_i^t - u_i^t \frac{\Delta t}{\Delta x}(\rho_i^t - \rho_{i-1}^t) - \rho_i^t \frac{\Delta t}{\Delta x}(u_i^t - u_{i-1}^t) \quad (18)$$

Corrector step (forward difference):

$$u_i^{P2} = u_i^t - \frac{1}{\rho_i^t}\frac{\Delta t}{\Delta x}(p_i^t - p_{i-1}^t) - u_i^t \frac{\Delta t}{\Delta x}(u_i^t - u_{i-1}^t) + \Delta t\left(\frac{\omega_i^t}{\rho_i^t A} + g\sin\theta\right) \quad (19)$$

$$p_i^{P2} = p_i^t - u_i^t \frac{\Delta t}{\Delta x}(p_i^t - p_{i-1}^t) - [a_i^t]^2 \rho_i^t \frac{\Delta t}{\Delta x}(u_i^t - u_{i-1}^t) + \Delta t\left[\left(1-[\delta_s]_i^t\right)\left(\frac{\Omega_i^t + \omega_i^t u_i^t}{A}\right)\right] \quad (20)$$

$$\rho_i^{C2} = \rho_i^{P2} - u_i^{P2}\frac{\Delta t}{\Delta x}(\rho_{i+1}^{P2} - \rho_i^{P2}) - \rho_i^{P2}\frac{\Delta t}{\Delta x}(u_{i+1}^t - u_i^{P2}) \quad (21)$$

$$u_i^{C2} = u_i^{P2} - \frac{1}{\rho_i^{P2}}\frac{\Delta t}{\Delta x}(p_{i+1}^{P2} - p_i^{P2}) - u_i^{P2}\frac{\Delta t}{\Delta x}(u_{i+1}^{P2} - u_i^{P2}) + \Delta t\left(\frac{\omega_i^{P2}}{\rho}A + g\sin\theta\right) \quad (22)$$

$$p_i^{C2} = p_i^{P2} - u_i^{P2}\frac{\Delta t}{\Delta x}(p_{i+1}^{P2} - p_i^{P2}) - [a_i^{P2}]^2 \rho_i^{P2}\frac{\Delta t}{\Delta x}(u_{i+1}^{P2} - u_i^{P2})$$

$$-\Delta t\left[\left(1-[\delta_S]_i^{P2}\right)\left(\frac{\Omega_i^{P2}+\omega_i^{P2}u_i^{P2}}{A}\right)\right] \qquad (23)$$

The value of each variable at the end of a time step is the average of the variables' values at the beginning of the time step and its corrected values.

$$\rho_i^{t+\Delta t} = \frac{1}{2}(\rho_i^t + \rho_i^{C1}) \quad \rho_i^{t+\Delta t} = \frac{1}{2}(\rho_i^t + \rho_i^{C2}) \qquad (24)$$

$$u_i^{t+\Delta t} = \frac{1}{2}(u_i^t + u_i^{C1}) \quad u_i^{t+\Delta t} = \frac{1}{2}(u_i^t + u_i^{C2}) \qquad (25)$$

$$p_i^{t+\Delta t} = \frac{1}{2}(p_i^t + p_i^{C1}) \quad p_i^{t+\Delta t} = \frac{1}{2}(p_i^t + p_i^{C2}) \qquad (26)$$

It is possible to use each of the two alternatives exclusively or alternatively at each time step. Another more complicated alternative is to use an average of the corrected values obtained when both alternatives are carried out at every time step. In equation form, this alternative is expressed as follows:

$$\rho_i^{t+\Delta t} = \frac{1}{2}\rho_i^t + \frac{1}{4}\rho_i^{C1} + \frac{1}{4}\rho_i^{C2} \qquad (27)$$

$$u_i^{t+\Delta t} = \frac{1}{2}u_i^t + \frac{1}{4}u_i^{C1} + \frac{1}{4}u_i^{C2} \qquad (28)$$

$$p_i^{t+\Delta t} = \frac{1}{2}p_i^t + \frac{1}{4}p_i^{C1} + \frac{1}{4}p_i^{C2} \qquad (29)$$

The merits, demerits and suitability for application of the three alternatives of using the MacCormack method were discussed [17]. The three alternatives were compared. However, it should be noted that it was recommended [22] to use the average values with both alternatives, in cases where it is important that the signal be transmitted at the correct speed.

VALIDATION OF THE COMPUTER MODEL

Results produced by the computer model were compared with experimental results from the Foothills tests [23]. In the Foothills test, short pipe lengths of a total of 243m and diameters of approximately 1.2 and 1.4m were charged with natural gas of known composition and pressurised to between 74 and 87 barA. Fracture was initiated at the centre of the test section by detonating an explosive cutter. Although the data can be used to some extent to validate computer models for linebreak analysis, it is not suitable for this purpose because the fracture was designed to propagate along the axial direction of the pipe covering some considerable lengths. This makes it difficult to model the break boundary, especially using this model where the break boundary is assumed to be fixed in the x-t plane. Only one test result, namely NABTF1 EAST, was selected for validation of the computer model. The result NABFT1 EAST was used to validate the model for flow reversal in the downstream section of the broken pipe. Results produced by the computer model are presented in Figs. 2 and 3, together with the experimental results and numerical results produced by the characteristics model [1]. A grid spacing of $x = 0.1\text{m}$ and $t = 0.0001\text{s}$, at the broken end and a variable grid spacing were used.

DISCUSSION OF THE VALIDATION OF RESULTS

The symbols used in the graphs are defined as follows: MOC (Method of characteristics) and MCC (MacCormack method). Results and observations made in using the method of characteristics were presented in preceding paper [1]. In addition, it was observed that in situations where there are sharp changes in the fluid properties, such as during the first few t' s in the region around the break, the second-order method of characteristics failed numerically. For such cases, the first-order method was used throughout the calculation. Also when the method of characteristics was used to model the flow reversal in the section of the pipeline downstream of the break, it produced results which tend to lean on the values at the intact end of the pipeline section. This directional bias resulted in a very slow pressure drop in the broken section of the pipeline, including the broken boundary. The problem of directional bias does not exist with the programme based on the MacCormack method. An investigation into the possible causes of the problem did not reveal any error in the calculation procedure or computer coding. The problem of directional bias with the method of characteristics was observed only when the second-order approximation was used. Both the upstream and downstream models

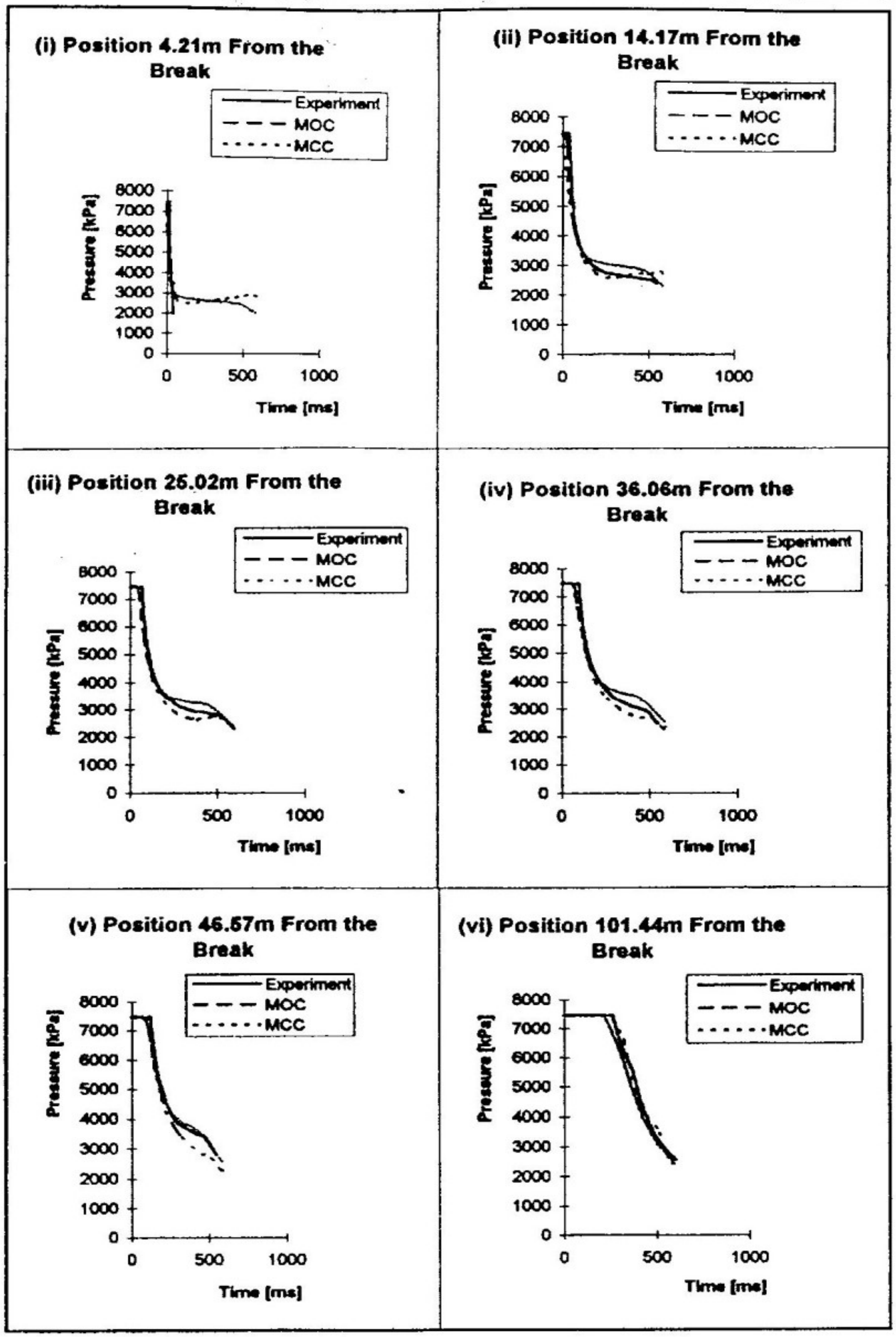

Fig. 2 p-t Curves for Foothills Test NABTF1 East

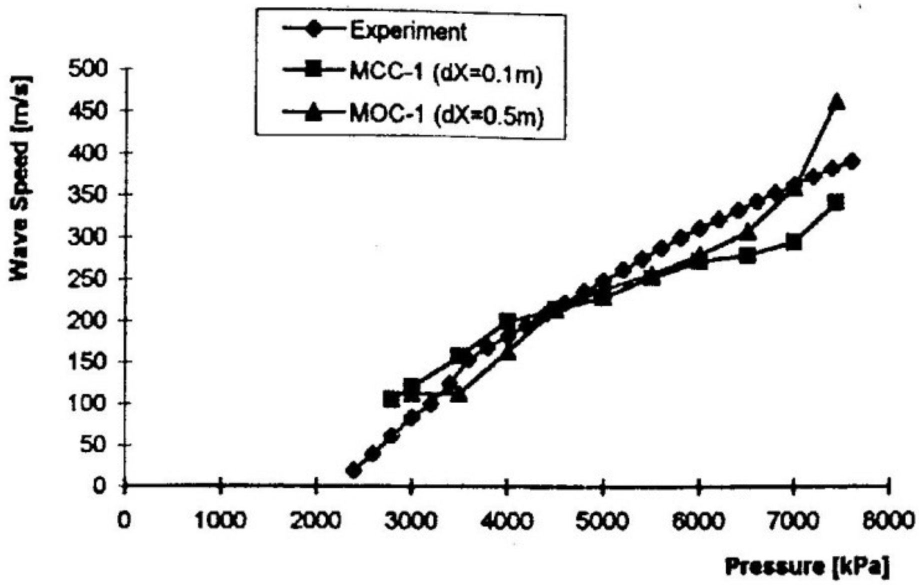

Fig.3 Pressure Wave Propagation Speed for Foothills Test NABTF1

produce comparable results, when the first-order method of characteristics was used.

It was earlier concluded [22] that the MacCormack method is superior to the method of characteristics when C_n differs appreciably from unity. When C_n is much smaller than unity, the MacCormack method produces results with a precision that could not be attained with any reasonable number of computation nodes, when the method of characteristic is used. It was also concluded that the use of the alternatives 1 and 2 in succession on time steps could introduce significant oscillations in the solution, especially where the basic equations are poorly approximated. Directional bias could be avoided by using exclusively one of the two calculation alternatives. The directional bias is important only when working with two space dimensions, in which case it was recommended that the average of both methods (alternative 3) be used. It was also claimed that doing so did seem to smear the shock slightly and the computation time was doubled.

In this study the computer programme is written in a way that any of the three alternatives could be used. This is despite the fact that it is was decided to use alternative 1 exclusively. Results from the three alternatives were compared [17], using a different set of experimental data called British

Gas BGT2 [24]. Due to limited computer memory, only the first-order calculation of the method of characteristics could be used at the boundary points, when using alternative 3. Alternatives 1 and 3 produced similar results. Alternative 2 produced the worst results, with much bigger oscillations and pressures falling fastest. The computation speed of alternative 1 is higher than that of alternative 3. The MacCormack method is extremely simple to programme, compared with the method of characteristics. The execution speed of the MacCormack method is faster than that of the method of characteristics but in this case, where the time used to calculate the fluid properties from the QUANT software constitutes the biggest proportion of the CPU time, the two methods have execution speeds which do not differ much. The same argument applies for the difference which is to be expected between alternative 3 and the other two alternatives of the MacCormack method.

It was stated earlier that in the presence of shocks, explicit finite-difference methods of higher than first order produce considerable overshoot and oscillatory systems. Results obtained from the MacCormack method were oscillatory especially near the broken end. The oscillations are more severe for smaller L/D values. Also an overshoot was observed in the flow velocity, at the node next to the boundary node at the break. The overshoot was controlled by limiting the magnitude of the flow velocity to that of the corresponding speed of sound. These problems were not encountered with the method of characteristics. With the MacCormack method the problem of directional bias which was encountered with the method of characteristics, in the section of the pipeline down stream the break, did not exist.

CONCLUSIONS

The transient analysis models based on the method of characteristics and the MacCormack method were compared based on accuracy, stability of results and computational economy. The main reason for including the MacCormack was to confirm findings that the method is suitable for modelling the transient flow following a break in high-pressure gas pipelines. The literature review which was conducted during this study indicated that this method could be more suitable for high-pressure gas linebreak applications than the method of characteristics. A previous study

[2] used the second-order method of characteristics, but concluded that better results could be obtained by using an alternative numerical method of solution. In a previous publication [25], the MacCormack method was recommended as the most suitable for linebreak problems.

Based on the comparison made in this study, it is concluded that the method of characteristics produces better results than the MacCormack method, for linebreak applications. The criteria used in comparing the two models is accuracy and stability of results, computer memory and CPU time requirements. The MacCormack method was found to be unsuitable for modelling transient flow following a linebreak in high-pressure natural gas pipelines. It produced oscillating results in the low pressure region, which resulted in p-t curves crossing each other. It predicts the wave speeds reasonably well in the low pressure region, but it underestimates it in the high pressure region. The magnitude of equalization pressure is slightly higher than that calculated with the method of characteristics. The computation speed of the MacCormack method is one and a half times faster than that of the second-order method of characteristics, when the alternatives 1 and 2 were used. When the alternative 3 was used, the computation speed of the MacCormack method was the same as that of the second order-method of characteristics

The MacCormack second-order method was previously thought of as being potentially better for linebreak problems than the method of characteristics. In contrast, this study has confirmed that the MacCormack method is less suitable for modelling of the flow following linebreak in high-pressure gas pipelines.

NOMENCLATURE

A	=	Cross-section area of pipe
a	=	Wave speed
C_n	=	Courant number
d	=	Pipe diameter
f_D	=	Darcy's friction factor
g	=	Gravitational acceleration
h	=	Specific enthalpy of gas
L	=	Length of pipeline

M_a	=	Mach number
p	=	Static pressure of gas
Pr	=	Prandtl number
t	=	Time
u	=	Flow velocity of gas
x	=	Horizontal distance along the pipe

Greek Symbols

λ	=	Gradient of the characteristic lines
γ_s	=	Isentropic gamma coefficient
Δ	=	Small change in the quantity
δ_s	=	Isentropic delta coefficient
θ	=	Angle of inclination of pipe to horizontal
μ	=	Coefficient of dynamic viscosity
ρ	=	Density of gas
ψ	=	Conical angle of the pipe
	=	Heat flow into the pipe per unit length of pipe per unit time
ω	=	Frictional force per unit length of pipe

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The manuscript was received on 23rd September 1997 and accepted for publication, after corrections, on 29th December 1997