COMPUTATION OF FREQUENCY CHARACTERISTICS OF LUMPED-ELEMENT 3 dB BRANCHLINE DIRECTIONAL COUPLERS

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ABSTRACT

An analysis of the 3 dB lumped-element branchline directional coupler constructed using L-C and T networks is given. Matrices of lumped element capacitances and inductances in the form of low and high pass filters are equated to the matrices of the distributed element transmission lines at the centre frequency of operation, as a result of which design equations are derived. Using even and odd modes of excitation, frequency characteristics of the directional couplers are computed and compared to those of the coupler constructed using distrtibuted element transmission lines.

1. INTRODUCTION

The 3 dB branchline directional coupler belongs to a group of hybrid networks which offer equal division of power at the two output ports but with 90° difference in their phases [Gardiner & Mgombelo, 1981]. From Figure 1, power incident on port 1 is split equally between ports 3 and 4 but with 90° difference in phase, ideally. Thus ports 1 and 2 are isolated and so are ports 3 and 4.

Hybrid networks are extensively used in communication circuits such as transmitters and antennas, The 3dB branchline directional coupler is very easy to design and manufacture and offers a considerable wide bandwidth of operation especially when several units are cascaded.

The branchline directional coupler can be realized using transmission lines such as coaxial cables, striplines, microstrip lines and also using lumped-element components [Beach et al., 1985]. The use of lumped elements is particularly attractive in the HF and VHF bands where the dimensions of the coupler constructed using distributed elements becomes too large.

This paper considers the latter case, whereby π and T networks consisting of inductances and capacitances are used instead of the transmission lines. Design equations are derived by equating the transmission (ABCD) matrices of the network with lumped elements to those of the transmission line at the centre frequency of operation. Performance characteristics are then calculated using even and odd modes of excitation over a 100% frequency bandwidth.

2. ANALYSIS

2.1 Design Relations

The different types of Π and T networks that can be used in the branchline hybrid are shown in Figure 2. Equating matrices of the Π and T networks to those of the quarterwavelength transmission lines at the centre frequency we get:

$$L_{H\Pi} = \frac{z_o}{\omega_o\left[-\tan\frac{\theta}{2}\right]} = \frac{z_o}{\omega_o}$$

$$C_{H\Pi} = \frac{1}{\omega_o z_o(-\sin\theta)} = \frac{1}{\omega_o z_o} \quad \ldots\ldots(1)$$

$$L_{HT} = \frac{z_o}{\omega_o(-\sin\theta)} = \frac{z_o}{\omega_o}$$

$$C_{HT} = \frac{1}{\omega_o z_o\left(-\tan\frac{\theta}{2}\right)} = \frac{1}{\omega_o\ z} \quad \ldots\ldots(2)$$

$$L_{L\Pi} = \frac{z_o\ \sin\theta}{\omega_o} = \frac{z_o}{\omega_o}$$

$$C_{L\Pi} = \frac{\tan\frac{\theta}{2}}{\omega_o\ z_o} = \frac{1}{\omega_o\ z_o} \quad \ldots\ldots(3)$$

$$L_{LT} = \frac{z_o\ \tan\frac{\theta}{2}}{\omega_o} = \frac{z_o}{\omega_o} \quad \ldots\ldots(4)$$

$$C_{LT} = \frac{\sin\theta}{\omega_o z_o} = \frac{1}{\omega_o z_o}$$

where

$\omega_o = 2\pi f_o$

f_o = centre frequency,

z_o = characteristic impedance of the equivalent transmission line,

$\theta = \beta l = 2\pi l/\lambda$ electrical length of the transmission line,

C_{XY} and L_{XY} are values of capacitance and inductance for

X = H for High Pass Filter,

X = L for Low Pass Filter,

and Y = Π for Π network,

Y = T for T network.

It is important to note that for the High Pass Filter cases in equations (1) and (2) above the value of the trigonometric function should be negative, so as to get realizable values of capacitances and inductances. This implies that the lumped elements have now to be equated to a 3λ/4 (and not λ/4 as in the earlier case) long transmission line, and henceforth added insertion loss. Below only the performance characteristics of the low pass filter networks are considered.

2.2 Performance Characteristics

For the sake of generalization normalized transmission (ABCD) matrices [Garver, 1976] will be used in this paper.

The normalized transmission matrix of a Π low pass filter L-C network (see Figure 2) equivalent to the transmission line between ports 1 and 3 with characteristic impedance $z_o/\sqrt{2}$ (Figure 1) can be easily found to be:

$$\begin{bmatrix} A_\Pi & B_\Pi \\ C_\Pi & D_\Pi \end{bmatrix} = \begin{bmatrix} 1-\alpha^2 & j\frac{\alpha}{\sqrt{2}} \\ j\alpha\sqrt{2}(2-\alpha^2) & 1-\alpha^2 \end{bmatrix} \qquad \text{......(5)}$$

while that for a T network is

$$\begin{bmatrix} A_T & B_T \\ C_T & D_T \end{bmatrix} = \begin{bmatrix} 1-\alpha^2 & j\frac{\alpha}{\sqrt{2}}(2-\alpha^2) \\ j\sqrt{2}\alpha & 1-\alpha^2 \end{bmatrix} \qquad \text{......(6)}$$

where $\alpha = \frac{\omega}{\omega_o} = \frac{f}{f_o}$ normalized frequency parameter

Figure 3 shows the realization of the branchline directional coupler using the Π LPF network together with even and odd networks found by bisecting the network [Reed & Wheeler, 1956].

The even mode network is simply a cascade of two shunt C_2 reactances together with the Π network as shown in Figure 3(a). The overall normalized ABCD matrix for even mode becomes:

$$\begin{bmatrix} A_e^{\Pi} & B_e^{\Pi} \\ C_e^{\Pi} & D_e^{\Pi} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ j\alpha & 1 \end{bmatrix} = \begin{bmatrix} 1-\alpha^2 & j\frac{\alpha}{\sqrt{2}} \\ j\alpha\sqrt{2}(2-\alpha^2) & 1-\alpha^2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ j\alpha & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1-\alpha^2\ \frac{\sqrt{2}+1}{\sqrt{2}} & j\ \frac{\alpha}{\sqrt{2}} \\ j\alpha\sqrt{2}\left[\sqrt{2}(1-\alpha^2)+2-\alpha^2-\frac{\alpha^2}{2}\right] & 1-\alpha^2\ \frac{\sqrt{2}+1}{\sqrt{2}} \end{bmatrix} \quad \ldots\ldots(7)$$

From Figure 3(b) the overall normalized ABCD matrix for the odd mode can be found to be:

$$\begin{bmatrix} A_o^{\Pi} & B_o^{\Pi} \\ C_o^{\Pi} & D_o^{\Pi} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -j\left(\frac{2-\alpha^2}{\alpha}\right) & 1 \end{bmatrix} \begin{bmatrix} 1-\alpha^2 & j\frac{2}{\sqrt{2}} \\ j\alpha\sqrt{2}(2-\alpha^2) & 1-\alpha^2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -j\left(\frac{2-\alpha^2}{\alpha}\right) & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+\sqrt{2} - \alpha^2 \left(\frac{\sqrt{2}+}{\sqrt{2}}\right) & j\,\frac{\alpha}{\sqrt{2}} \\ j\left(\frac{2-\alpha^2}{\alpha}\right)\left[\alpha^2\sqrt{2}-2(1-\alpha^2)\ \ \frac{2-\alpha^2}{2}\right] & 1+\sqrt{2}-\alpha^2\ \frac{\sqrt{2}+1}{\sqrt{2}} \end{bmatrix} \quad \ldots\ldots(8)$$

Figure 4 shows the realization of the branchline directional coupler using the T-LPF network together with the even and odd mode networks.

The normalized ABCD matrix for the even mode becomes

$$\begin{bmatrix} A_e^T & B_e^T \\ C_o^{\Pi} & D_e^T \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ j\left(\frac{\alpha}{2-\alpha^2}\right) & 1 \end{bmatrix} \begin{bmatrix} 1-\alpha^2 & j\frac{\alpha}{\sqrt{2}}(1-\alpha^2) \\ j\ \sqrt{2}\alpha & 1-\alpha^2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \left(\frac{\alpha}{2-\alpha^2}\right) & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+\sqrt{2} - \alpha^2\left(\frac{\sqrt{2}+1}{\sqrt{2}}\right) & j\frac{\alpha}{\sqrt{2}}(1-\alpha^2) \\ j\left(\frac{1}{2\alpha^2}\right)\left[\frac{2\sqrt{2}+4-\alpha^2(2\sqrt{2}+3)}{\sqrt{2}}\right] & 1-\alpha^2\ \frac{\sqrt{2}+1}{\sqrt{2}} \end{bmatrix} \quad \ldots\ldots(9)$$

Similarly for the odd mode we get

$$\begin{bmatrix} A_o^T & B_o^T \\ C_o^{\Pi} & D_o^T \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -j\frac{1}{\alpha} & 1 \end{bmatrix} \begin{bmatrix} 1-\alpha^2 & j\frac{\alpha}{\sqrt{2}}(1-\alpha^2) \\ j\sqrt{2}\alpha & 1-\alpha^2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -j\frac{1}{\alpha} & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1-\alpha^2 + \frac{2-\alpha^2}{\sqrt{2}} & j\frac{\alpha}{\sqrt{2}}(1-\alpha^2) \\ j\frac{1}{\sqrt{2}\alpha}\,[\alpha^2(2\sqrt{2}+3) - 2(1+\sqrt{2})] & 1-\alpha^2 + \frac{2-1}{\sqrt{2}} \end{bmatrix} \quad .....(10)$$

Having found the normalized ABCD matrices for even and odd modes for two different realizations, we can easily compute the scattering parameters of the networks from which the performance characteristics can be derived.

The following relations have to be used [Garver, 1976]:

$$S_{11}^{i} = \frac{A_i + B_i - C_i - D_i}{A_i + B_i + C_i + D_i}\,, \quad .....(11)$$

$$S_{12}^{i} = \frac{1}{A_i + B_i + C_i + D_i} = S_{21}^{i}\,, \quad .....(12)$$

$$S_{22}^{i} = \frac{-A_i + B_i - C_i + D_i}{A_i + B_i + C_i + D_i}, \qquad \text{.....(13)}$$

where the subscript is used to denote the mode of excitation (i = e for even mode and i = 0 for odd mode).

The different scattering matrix parameters can be found from, the following relations

$$S_{11} = \frac{1}{2}\left[S_{11}^{e} \quad S_{11}^{o}\right] = S_{22}, \qquad \text{.....(14)}$$

$$S_{12} = \frac{1}{2}\left[S_{11}^{e} \quad S_{11}^{o}\right] = S_{21}, \qquad \text{.....(15)}$$

$$S_{13} = S_{31} = S_{24} = S_{42} = \frac{1}{2}\left[S_{12}^{e} - S_{12}^{o}\right], \qquad \text{.....(16)}$$

$$S_{14} = S_{41} = S_{32} = S_{23} = \frac{1}{2}\left[S_{12}^{e} - S_{12}^{o}\right]. \qquad \text{.....(17)}$$

For the directional coupler the following performance characteristics can be found:

Input VSWR at port 1

$$VSWR_{in} = \frac{1 + |S_{11}|}{1 - |S_{11}|} \; . \qquad .....(18)$$

Isolation between ports 1 and 2 (or between ports 3 and 4)

$$I = 20 \log_{10}(1/|S_{12}|)dB. \qquad .....(19)$$

Insertion loss or split from port 1 to port 3

$$S = 20 \log_{10}(1/|S_{13}|)dB. \qquad .....(20)$$

Coupling factor from port 1 to port 4

$$C = 20 \log_{10}(1/|S_{14}|)dB. \qquad .....(21)$$

Equations (18)-(21) have been used to plot the frequency characteristics of the two types of realizations, as shown in Figures 5-8. For comparison purposes, the characteristics of the directional coupler constructed using distributed element transmission lines are also shown.

3. CONCLUSIONS

The analysis of the 3 dB lumped-element branchline directional coupler has been presented in this paper. The matrices of the Π and T L-C networks have been equated to those of the transmission lines used in the coupler at the center frequency to give design equations. Even and odd modes of excitation have been used in calculating the performance characteristics of the realizations

of the coupler, one using a Π low pass filter L-C network and the other using a T low pass filter L-C network.

It is found that the lumped element realizations offer narrow bandwidths of equation as compared to the realization using distributed elements. This means that lumped-element realizations will find use in narrowband systems where the size of the devices is required to be as small as possible, for example in the HF and VHF bands.

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4. REED, and WHEELER, G.J., 1956, A method of analysis of symmetrical four-port networks, IRE,s Trans. Microwave Theory & Tech., Vol. MTT-4, pp. 246-252.

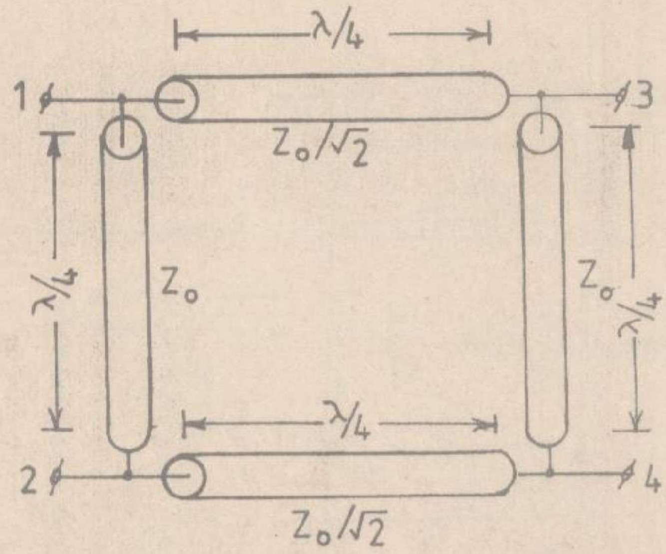


Fig. 1. 3-dB branchline directional coupler

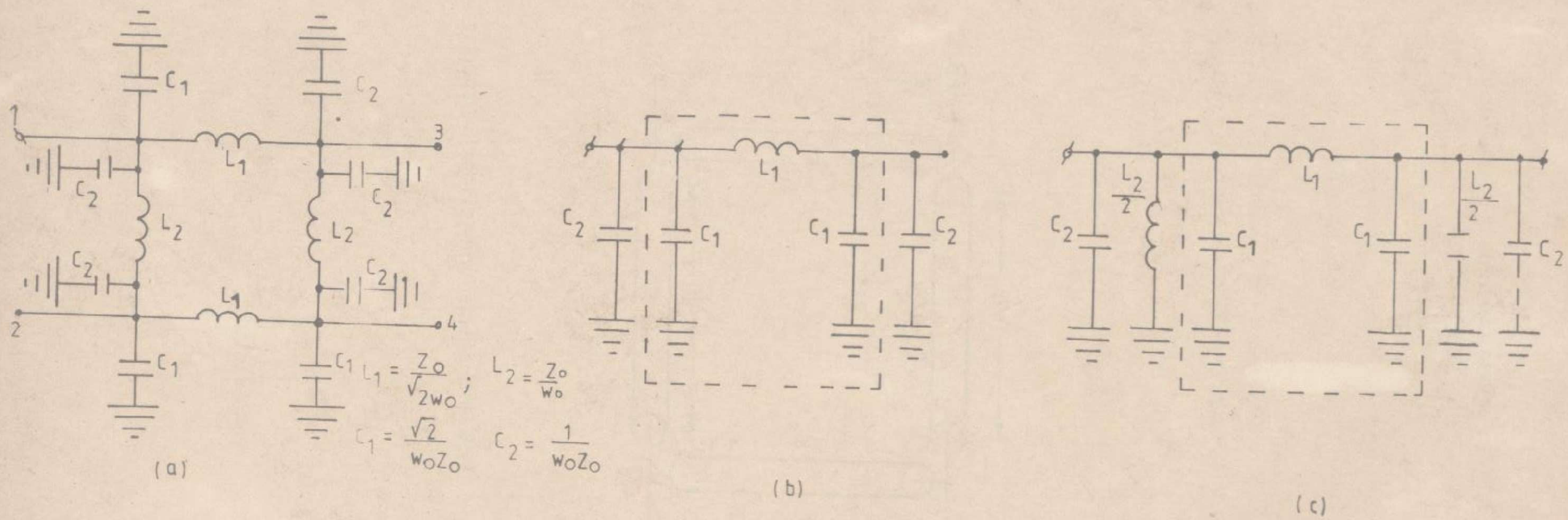


Fig. 3.Π - Low pass filter realisation of the branch line directional coupler
(a) network (b) even-mode network and (c) odd-mode network

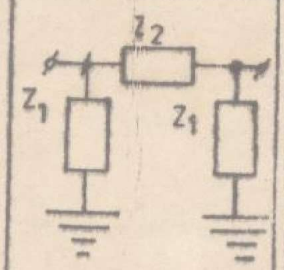
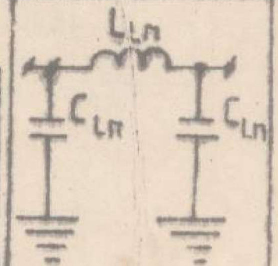
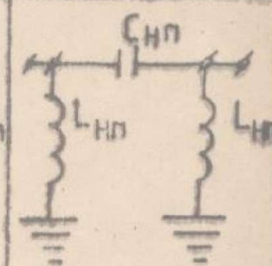
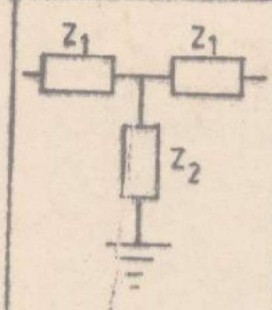
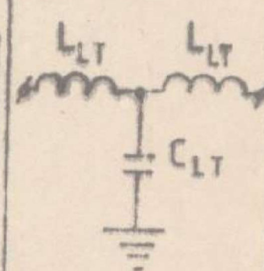
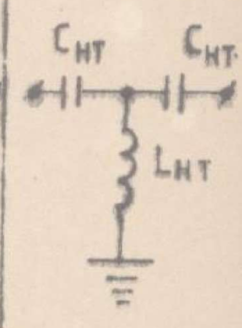
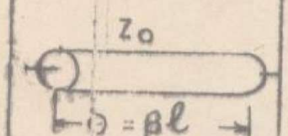
NET WORK NAME	NET WORK DIAGRAM	ABCD matrix	Low Pass Filter Realisation	High Pass Filter Realisation
Π		$\begin{bmatrix} 1+Z_2/Z_1 & Z_2 \\ \frac{2}{Z_1}+\frac{Z_2}{Z_1^2} & 1+\frac{Z_2}{Z_1} \end{bmatrix}$		
T		$\begin{bmatrix} 1+\frac{Z_1}{Z_2} & 2Z_1+\frac{Z_1^2}{Z_2} \\ \frac{1}{Z_2} & 1+\frac{Z_1}{Z_2} \end{bmatrix}$		
Transimission Line		$\begin{bmatrix} \cos\Theta & jZo\sin\Theta \\ j\frac{\sin\Theta}{Z_o} & \cos\Theta \end{bmatrix}$		

Fig. 2. Π and T networks for use in the branchline directional coupler

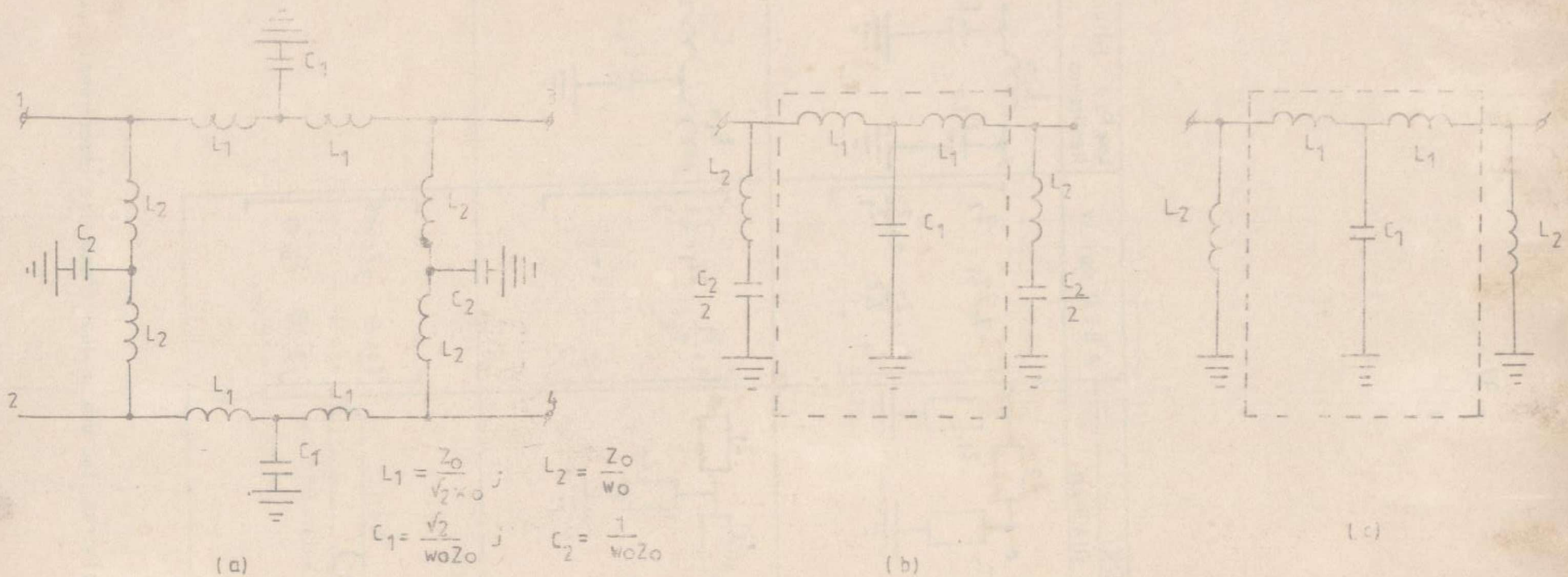


Fig. 4. T-low pass filter realisation of branch line directional coupler
(a) network (b) even mode network and (c) odd mode network

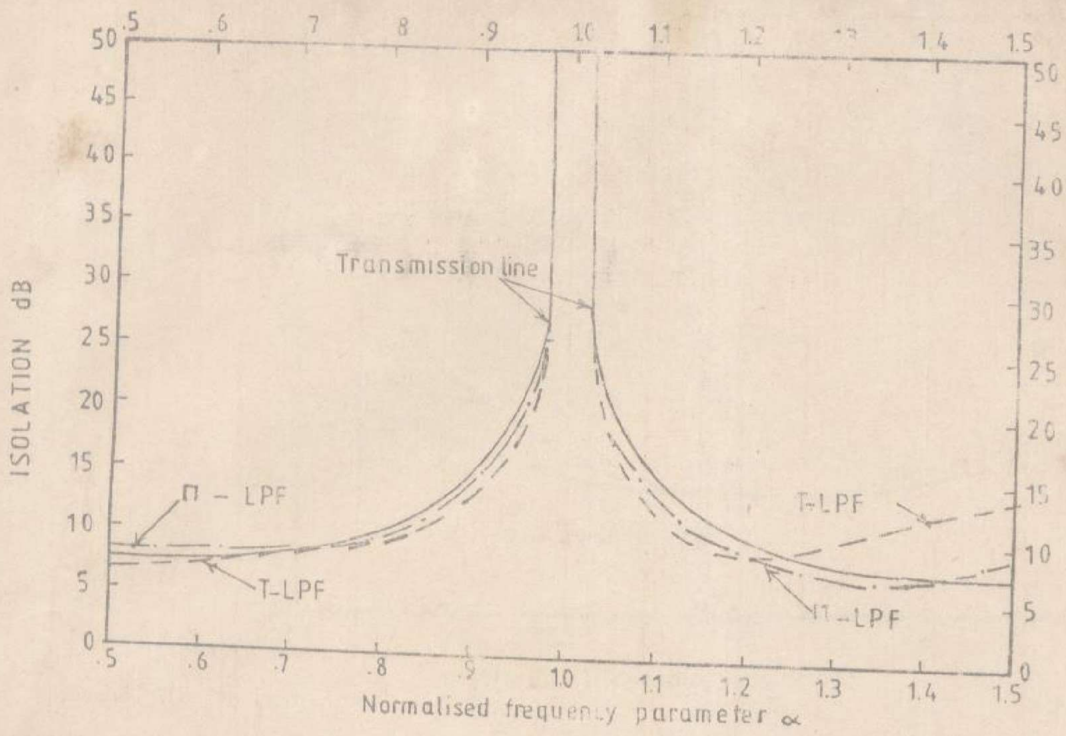


Fig. 6. Isolation characteristics

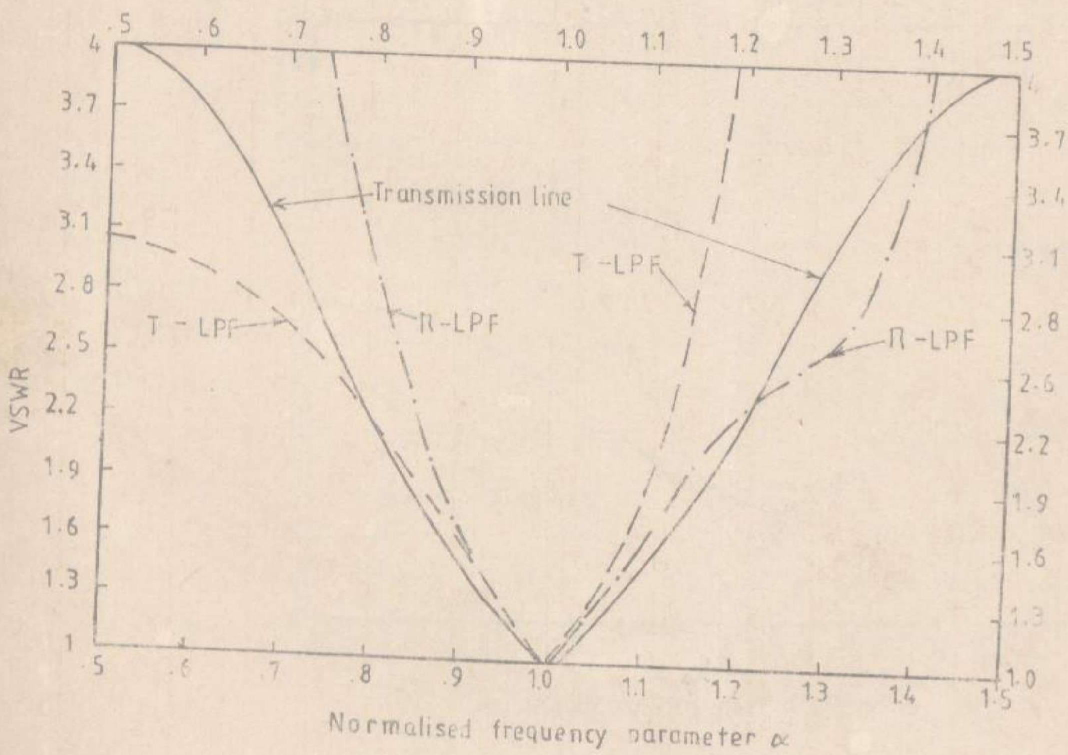


Fig. 5. VSWR characteristics

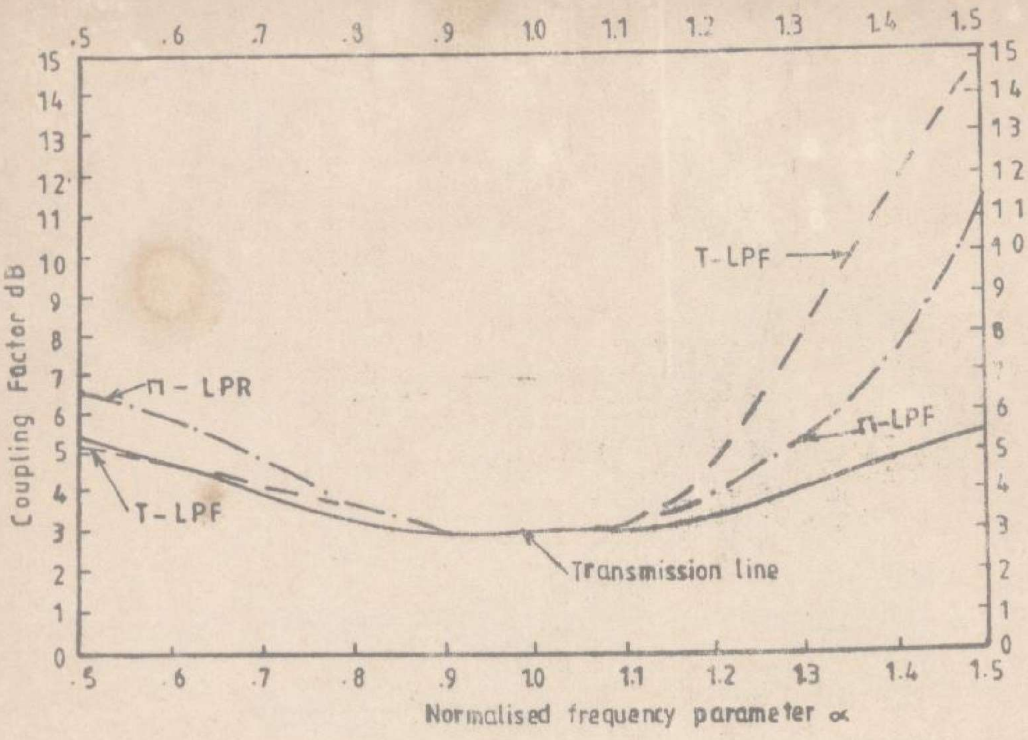


Fig 8 Coupling factor characteristics

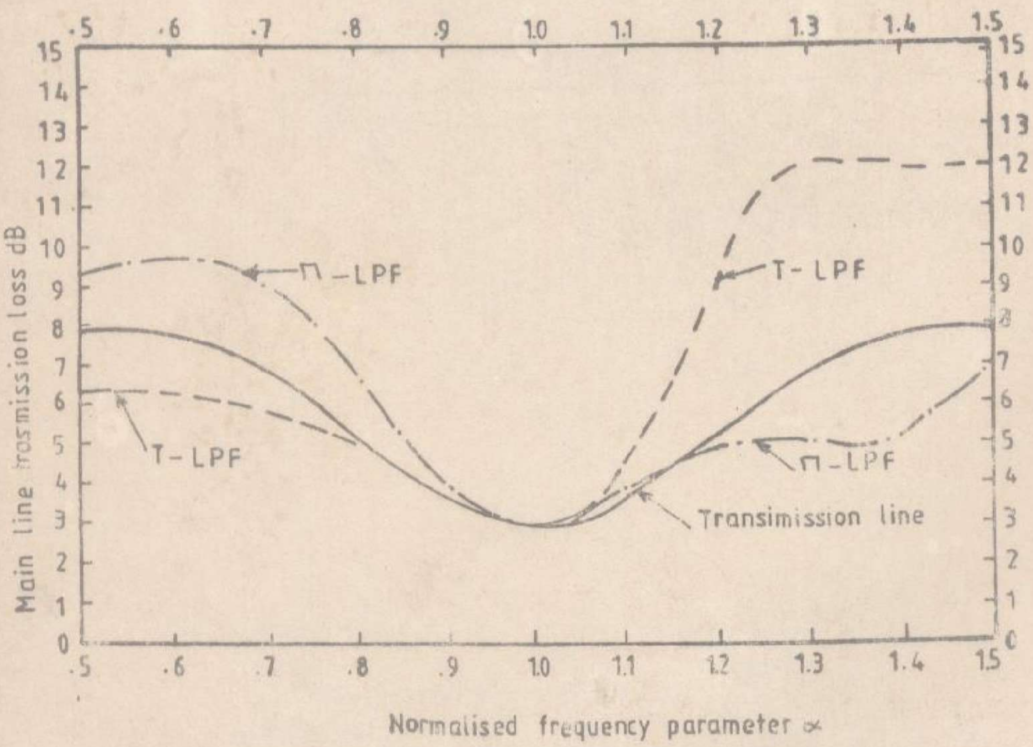


Fig. 7. Main line transmission loss characteristics