EFFECT OF UNBALANCED VOLTAGE ON OPERATION OF POLYPHASE INDUCTION MOTORS PART II

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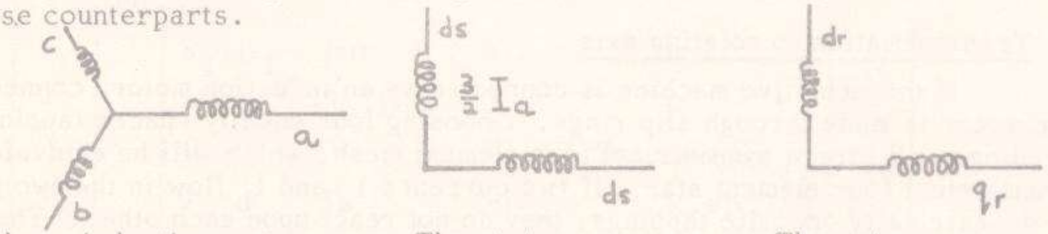
Abstract

The general theory of polyphase induction motors was published by UHANDISI Journal Vol. 4 of June 1978. Part II of this topic deals with the actual operation of polyphase induction motors due to the unbalanced voltages. In this paper an analytical method has been developed using Kron's transformation matrix. This method transforms a three phase motor to a two-phase motor. The two-phase motor (the primitive machine) will have two stator windings at right angles and two equivalent rotor windings also at right angles. This type of primitive machine will represent a three phase induction motor. The final paper will give test results and it is hoped that it will be published by UHANDISI as part III.

1. Transformation of three-phase to a two-phase

The primitive machine as it is usually known will have two stator windings at right angles and two equivalent rotor windings at right angles and two equivalent rotor windings also at right angles. This type of machine will represent a three phase induction motor.

Figure 1 shows the three-phase windings converted into their equivalent two-phase counterparts.



Three-phase induction motor The stator The roto

Figure 1: Three-phase to two-phase transforms

The rotating field produced by three-phase currents in a three-phase stator can equally well be produced by two-phase currents in a two-phase winding. Figure 1 shows coils a, b and c of three-phase windings distributed in space of 120° el. degrees to each other and also the same coils redistributed to form a two-phase winding each having 3/2 times as many coils.

Equating mmf components in the direct and quadrature axes we have:

$$3/2\ I_{qs} = I_a - 1/2 I_b - 1/2 I_c \quad \text{.............................} \quad (1)$$

$$3/2\ I_{ds} = 0 + \frac{\sqrt{3}}{2} I_b + \frac{\sqrt{3}}{2} I_c \quad \text{.............................} \quad (2)$$

$$I_o = 1/3(I_a + I_b + I_c) \quad \text{.............................} \quad (3)$$

or in matrix form

$$\begin{bmatrix} I_{qs} \\ I_{ds} \\ I_o \end{bmatrix} = 2/3 \begin{bmatrix} 1 & -1/2 & -1/2 \\ o & -\frac{\sqrt{3}}{2} & \frac{\sqrt{3}}{2} \\ 1/2 & 1/2 & 1/2 \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \quad \text{...................} \quad (4)$$

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or

$$\begin{bmatrix} I_{qs} \\ I_{ds} \\ I_o \end{bmatrix} = \begin{bmatrix} C \end{bmatrix} \cdot = \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \qquad (5)$$

where $[C]$ is the transfer matrix.

Now finding the original three-phase balanced currents in terms of two-phase balanced sequence currents we have:

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} C \end{bmatrix}^{-1} \begin{bmatrix} I_{qs} \\ I_{ds} \\ I_o \end{bmatrix} \qquad (6)$$

After finding the inverse transfer matrix C we ha ve

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ -1/2 & \frac{\sqrt{3}}{2} & 1 \\ -1/2 & \frac{\sqrt{3}}{2} & 1 \end{bmatrix} \begin{bmatrix} I_{qs} \\ I_{ds} \\ I_o \end{bmatrix} \qquad (7)$$

in a similar manner we can calculate the inverse transfer matrix for voltages.

2. <u>Transformation to rotating axis</u>

If the primitive machine is connected as an induction motor, connection to the motor is made through slip rings. Choosing four equally spaced tappings, the windings will form a symmetrical four element mesh, which will be equivalent to a symmetrical four element star. If two current s I_a and I_b flow in the two pairs of diametrically opposite tappings, they do not react upon each other. The motor can thus be represented by two separate coils a and b at right angles set in a frame of reference which rotates relative to the fixed axes d and q (see fig. 2). Currents will as well be transformed to correspond with currents in the fixed frame

Resolving the components I_{ar} and I_{br} along the two axes and equating to I_{dr} and I_{qr} respectively we obtain

$$\begin{bmatrix} I_{dr} \\ I_{qr} \end{bmatrix} = \begin{bmatrix} \cos\varphi & -\sin\varphi \\ \sin\varphi & \cos\varphi \end{bmatrix} \begin{bmatrix} I_{ar} \\ I_{qr} \end{bmatrix} \qquad (8)$$

Taking the inverse of the transfer matrix we have:

$$\begin{bmatrix} I_{ar} \\ I_{br} \end{bmatrix} = \begin{bmatrix} \cos\varphi & \sin\varphi \\ -\sin\varphi & \cos\varphi \end{bmatrix} \begin{bmatrix} I_{ar} \\ I_{qr} \end{bmatrix} \qquad (9)$$

In a similar manner the inverse transfer matrix for voltages can be found

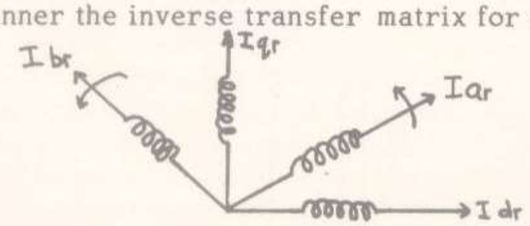

Fig. 2: Resolving the components I_{ar} and I_{br} along the two fixed axes

3. Components of Stator and Rotor Currents

Assuming that both rotor and stator currents of the induction motor are unbalanced, then systems should be converted to their symmetrical components.

$$\begin{bmatrix} I_{ds} \\ I_{dr} \\ I_{qr} \\ I_{qs} \end{bmatrix} = \begin{bmatrix} C \end{bmatrix} \begin{bmatrix} I_{if} \\ I_{2f} \\ I_{2b} \\ I_{1b} \end{bmatrix} \quad \ldots\ldots (10)$$

where

$$\begin{bmatrix} C \end{bmatrix} = \frac{1}{V_2} \begin{bmatrix} 1 & o & o & 1 \\ o & 1 & 1 & 1 \\ o & -j & j & o \\ -j & o & o & j \end{bmatrix} \quad \ldots\ldots (11)$$

The relationship between the voltages and currents is given by the following equation

$$\begin{bmatrix} V^1 \end{bmatrix} = \begin{bmatrix} C_T^* \end{bmatrix} \begin{bmatrix} Z \end{bmatrix} \begin{bmatrix} C \end{bmatrix} \begin{bmatrix} I^1 \end{bmatrix} \quad \ldots\ldots (12)$$

where

$\begin{bmatrix} C_T^* \end{bmatrix} \begin{bmatrix} Z \end{bmatrix} \begin{bmatrix} C \end{bmatrix}$ is the impedance matrix of the induction motor. Complete solution for equation 12 is given in Appendix 1. The impedance matrix is therefore given by:

$$\begin{bmatrix} V_f \\ 0 \\ 0 \\ V_n \end{bmatrix} = \begin{bmatrix} R_1+jx_1 & jxm & 0 & 0 \\ jsxm & R_2+jsx_2 & 0 & 0 \\ 0 & 0 & R_2+j(2-S)x_2 & j(2-S)x_m \\ 0 & 0 & -jx_m & R_1+jx_1 \end{bmatrix} \begin{bmatrix} I_{if} \\ I_{2f} \\ I_{2b} \\ I_{1b} \end{bmatrix} \quad \ldots\ldots (13)$$

The impendance matrix shows no mutual terms connecting forward and backward components and hence we may infer that the two components do not interest. Figure 3 corresponds to equation 13.

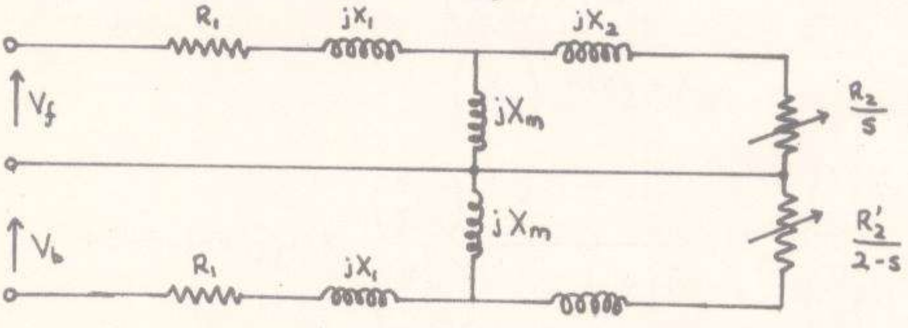

Fig.3: Equivalent circuit of an induction motor

In figure 3 V_f is responsible for the forward component of the current in the top part of the diagram and V_b is responsible fo backward components in the lower part.

4. Calculation of Sequence impedances

The sequence impedances are determined from the equivalent circuits in Figure 4. The values of R_1 and X_2 must correspond to the low frequency S_{f1} for the positive sequence, R'_2 and X_2 must correspond to a frequency of $(2-S)_{f1}$ for the negative sequence.

$$I_{Af} = I^+_{Af} + I^-_{Af}$$

$$I_{Bf} = I^+_{Bf} + I^-_{Bf} = I^+_{Af}\angle 240^\circ + I^-_{Af}\angle 120^\circ \quad \ldots\ldots(18)$$

$$I_{Cf} = I^+_{Cf} + I^-_{Cf} = I^+_{Af}\angle 120^\circ + I^-_{Af}\angle 240^\circ$$

7. Stator losses with unbalanced line currents

The total stator I^2R losses are equal to the stator resistance per phase, R_1 multiplied by the sum of the squares of the line currents, or

$$(\text{Stator } I^2R) = (I^2_{Af} + I^2_{Bf} + I^2_{Cf})R_1 \ldots\ldots(19)$$

Equation 19 shows that the total stator I^2R losses are equal to the sum of the losses produced by the positive and negative currents acting independently.

8. The total negative sequence rotor losses

The negative sequence electric input power to the rotor is equal to

$$3(I^-_{Af})^2\left(R_1 + \frac{\left(\frac{R'_2}{2-S}\right) X_m^2}{\left(\frac{R'_2}{2-S}\right)^2 + (X_2 + X_m)^2} - R_1\right) \ldots\ldots(20)$$

An additional amount of power is subtracted from the mechanical output and converted to rotor loss. This power is equal to:

$$(2-S)3(I^-_{Af})^2\left(R_1 + \frac{\left(\frac{R'_2}{2-S}\right) X_m^2}{\left(\frac{R'_2}{2-S}\right)^2 + (X_2 + X_m)^2} - R_1\right) \ldots\ldots(21)$$

Therefore the total rotor loss

$$\text{Rotor loss} = 3(I^-_{Af})\left(R_1 + \frac{\left(\frac{R'_2}{2-S}\right) X_m^2}{\left(\frac{R'_2}{2-S}\right)^2 + (X_2 + X_m)^2} - R_1\right) +$$

$$+(2-S-1)3(I^-_{Af})\left(R_1 + \frac{\left(\frac{R'_2}{2-S}\right) X_m^2}{\left(\frac{R'_2}{2-S}\right)^2 + (X_2 + X_m)^2}\right) = (I^-_{Af})^2 \frac{\left(\frac{R'_2}{2-S}\right) X_m^2}{\left(\frac{R'_2}{2-S}\right)^2 + (X_2+X_m)^2}(2-S) \ldots(22)$$

9. Conclusion

The above discussions on the effect of unbalanced voltages to the operation of induction motors implies that1 when unbalanced voltages are applied or occur to three or to a polyphase induction motor, excessive heating is experienced. Such heat is attributed to many factors such as:

a) The negative sequence currents produce an increased total copper loss in both rotor and stator and a decrease in net-shaft torque for a given positive sequence current.

b) The unbalanced currents produce an unbalanced spartial distribution of heating in the stator whicle core loss increases a negligible amount.

Since the ratio of rotor current to stator, and the rotor resistance are greater for the negative sequence component, the negative sequence currents produce more heat per ampere than the positive sequence currents. In addition to the thermal effects listed above, negative sequence components cause vibration that may be injurious to bearings, insulation and to interconnecting mechanical equipment. The nuisance of increased noise also results.

(In Part III which will be the final paper, test results will be discussed).

Appendix

The relationship between the voltages and currents is given by solving equation 12 in the following way:

$$[V'] = [C_T^*][Z][C][I']$$

$$[C_T^*][Z][C]$$

$$[C_T^*][Z] = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 0 & 0 & j \\ 0 & 1 & j & 0 \\ 0 & 1 & -j & 0 \\ 1 & 0 & 0 & -j \end{bmatrix} \begin{bmatrix} R_1+jX_1 & jX_m & 0 & 0 \\ jX_m & R_2+jX_2 & (1-s)X_2 & (1-s)X_m \\ -(1-s)X_m & -(1-s)X_2 & R_2+jX_2 & jX_m \\ 0 & 0 & jX_m & R_1+jX_1 \end{bmatrix}$$

$$= \frac{1}{\sqrt{2}} \begin{bmatrix} R_1+jX_1 & jX_m & -X_m & j(R_1+jX_1) \\ jX_m-j(1-s)X_m & R_2+jX_2-j(1-s)X_2 & (1-s)X_2+j(R_2+jX_2) & (1-s)X_m+X_m \\ jX_m-j(1-s)X_m & R_2+jX_2+j(1-s)X_2 & (1-s)X_2-j(R_2+jX_2) & (1-s)X_m+X_m \\ R_1+jX_1 & -jX_m & X_m & -j(R_1+jX_1) \end{bmatrix}$$

The continuation of the Appendix is available upon request from the Editors.

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