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Mathematical Model for Tanzania Population Growth

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Abstract

In this paper, a mathematical model for Tanzania population growth is presented. The model is developed by using exponential and logistic population growth models. Real data from censuses conducted by Tanzania National Bureau of Statistics (NBS) are used. The Tanzania growth rate is obtained by using data of 1967 and 2012 censuses. The prediction of population for the period of 2013 to 2035 is done. Numerical results show that the population grows at the rate of 2.88%. In 2035 the population is expected to be 87,538,767 by exponential model and 85,102,504 by logistic model. The carrying capacity is 2,976,857,550, which implies that the population will still grow faster since it is far from its limiting value. Comparisons of the models with real data from the five censuses are done. Also NBS projections are compared with populations predicted by the two models. Both comparisons show that the exponential model is performing better than logistic model. Also, the projection up to 2050 gives the population of 135,244,161 by exponential model and 131,261,794 by logistic model.

Keywords: Population growth, Exponential model, Logistic model, Carrying Capacity.

Introduction

The size of a population is an important parameter for economic development. The growth of population increases demands for food, water, energy and other economic needs. The growth of the population also determines the demand for essential social services, such as education, health, transport and housing (NBS 2006).

Knowing the future population is a core factor for development planning and decision making. Population projection provides future estimates of population size needed by planners and policy makers. Projections contribute to the improvement of quality of life of citizens through the provision of current and reliable data for policy formulation, development planning and service delivery as well as for monitoring and evaluating national and international development frameworks (NBS 2012).

Ofori et al. (2013) asserted that the major issue of the World is tremendous growth of the population especially in developing countries in relation to services provision. Population size and growth in a country can influence directly the situation of economy, policy, culture, education and environment (Yahaya et al. 2017).

Many countries use census to get the actual number of their population. The process is very costly; it involves a lot of people and technologies. Consequently, based on the cost incurred, census cannot be conducted in short periods of time necessitating the use of projection models to predict the future population.

Several mathematical population models have been developed and applied.

Mathematical models are very useful for projecting the total population. That is, the past data of the population is used to predict the future population. The most popular and applied models are exponential and logistic.

The exponential growth model was proposed by Malthus in 1798 and is also called the Malthusian growth model (Malthus 1798). The model suggests that the growth is exponential and has no bound. This model is widely regarded in the field of population ecology as the principle of population dynamics (Ashraf and Galor 2008, Wali et al. 2012).

Logistic growth model was proposed by Verhulst in 1845 (Marsden et al. 2003). The model is the extension of Malthusian model. The main feature of logistic model is the inclusion of the limiting value which is the size of population that an environment can support. This value is called carrying capacity K. In this model, population increases faster when the number of people is very small compared to the value of carrying capacity (K) and slowly as the population size is near to the value of carrying capacity.

According to Matintu (2016), as the population increases, the environment ability to support population decreases and hence the population reaches the stability point, that is, the carrying capacity. Therefore, it is reasonable to consider the mathematical model which explicitly incorporates the idea of carrying capacity. Both models originated from observations of biological reproduction processes (Wei et al. 2015). Most of the successful models are based on the extended forms of these models.

Literature Review

Several studies have been conducted on mathematical population models. Ali et al. (2015) studied census data and predicted population of Bangladesh by using logistic model. They used a curve fitting method and tried to compare the prediction between the case when carrying capacity is known and when it is not known. They used a curve fitting method for the population of a certain period of consecutive 11 years. The method gave better predictions which were close to the actual values of population.

Ofori et al. (2013) developed mathematical model for Ghana's population growth. They applied the exponential and logistic growth models to project the population. Their study found that the exponential model gave a better projection compared to the logistic model. However, Ghana’s population properties are different from Tanzania, including growth rate, initial population and therefore carrying capacity estimations.

Kulkarni et al. (2014) estimated the population growth of India from 2009 to 2012 using the logistic model approach and gave a comparison with actual population of India for the same time period. An error equation was also deduced based on the trend line for the specified time period and the population of India for the year 2013 and onwards till 2025 had been estimated. The time required for India to reach its carrying capacity was also discussed.

Wali et al. (2012) also studied the application of logistic equation to model the population growth of Uganda. They used least squares method to compute the growth rate, the carrying capacity and the year when the population of Uganda will approximately be half of the carrying capacity. More works in population modeling are found in Dawed et al. (2014), Shepherd and Stojkov (2005), Law et al. (2003), Wali et al (2011) on Rwanda) and many others.

Few studies concerning Tanzania population have been conducted. For example, Madulu (2004) discussed the linkages between population growth and environmental degradation in Tanzania. Tanzania's major environmental problems, demographic characteristics, and the linkages between environmental change and rapid population growth both at national and regional levels were discussed. A conclusion made is that, environmental change is as an important factor to demographic and economic factors as it is population growth to economic development and environmental conservation.

In this paper, the mathematical model for Tanzania population growth is studied. This model is adapted from the presented models in the literature but applies actual data from NBS which is mostly used by the Government for planning. This differs with most models in literature which use standard data from the International Data Base (IDB), United Nations and World Bank (Ogoke and Nduka 2016, Wali et al. 2012). The main objective of this work is to develop a population model by using mathematical modeling which is less costly and can be used for predicting future population.

Tanzania population

Tanzania is a relatively large country in East Africa. It shares boarders with Kenya, Uganda, Rwanda, Zambia, Malawi, Mozambique, Burundi, Democratic Republic of Congo and also the Indian Ocean in the Eastern part. By area, Tanzania is the 31st largest country in the World and 13th largest country in Africa (out of 54 countries of Africa) with the area of 947,303 square kilometres. Tanzania is estimated to be among the countries with highest birth rates in the World. According to DESA, UN (2017), by 2017 Tanzania was the 24th country with largest population out of 197 countries of the World.

Moreover, Unies (2017) stated that more than half of the global population between 2015 and 2050 is expected to occur in Africa. During 2050, half of the World population growth is expected to be concentrated in nine counties which are India, Nigeria, Pakistan, Democratic Republic of Congo, Ethiopia, Republic of Tanzania, United States of America, Indonesia and Uganda.

The Tanzanian government has so far conducted five censuses. The first census was in 1967 followed by the censuses of 1978, 1988, 2002 and 2012. These censuses contributed to the improvement of quality of life through provision of current and reliable data. The table below displays the population with respect to the year of the census.

Table 1: Tanzania population from five censuses

Year	Population
1967	12,313,469
1978	17,512,610
1988	23,174,335
2002	34,443,603
2012	44,928,923

Table 1 shows that the population in 45 years has increased rapidly to more than 30 million people. Despite the high population, the country is still sparsely populated, that is, the density is high in some parts of the country and has been increasing over time (Madulu 2005). The Government of the United Republic of Tanzania adopted the National Population Policy in 1992. The policy encompasses the following items: population and development planning; equality, equity and social justice; reproductive health; natural resources; food production; information and databases, and advocacy (URT 2006).

The policy provides a framework and guidelines for integration of population variables into the development processes. Clearly the policy has an implication into population growth as it provides harmonious relationship between population and socio-economic dynamics.

Projections by National Bureau of Statistics (NBS)

Due to the fact that conducting census in short periods is so expensive, NBS projected the population of Tanzania for the period from 2013 to 2035 at a National level. These projections are currently used by the Government as the actual population for development planning. Cohort component technique was used for projecting which is the Rural Urban Projection program (RUP) that was developed by the US census bureau and spectrum system which is a computer package (NBS 2018). The projections show that the Tanzania population growth rate will decrease from 3.1 percent in 2013 (with the expected population of 46,356,279) to 2.8 percent in 2035 (with the expected population of 89,204,781). The projections are shown in Table 2.

Table 2: Tanzania population projections by NBS

Year	Population
2013	46,356,279
2014	47,831,361
2015	49,359,408
2016	50,941,672
2017	52,554,628
2018	54,199,163
2019	55,890,747
2020	57,637,628
2021	59,441,988
2022	61,280,743
2023	63,150,477
2024	65,070,448
2025	67,036,280
2026	69,054,148
2027	71,106,934
2028	73,197,399
2029	75,344,775
2030	77,537,166
2031	79,776,176
2032	82,057,431
2033	84,380,103
2034	86,766,463
2035	89,204,781

The projections show that the population will be 61,280,743 in 2022 when the sixth census is expected to be conducted. By 2035 the projections show the population of 89,204,781. The projections show the rapid population growth of total of more than 44 million people from 2012 to 2035, which is faster compared to the growth of the period of 45 years, that is, from 1967 to 2012.

Methodology and Data

In this paper, Tanzania population projections are done by using the exponential and logistic models. Data from the first census of 1967 is used which is different from NBS projections data that is only based on 2012 census. The projections of this study consider only the two important factors which influence population growth, these are, mortality and birth rates. Several previous censuses have shown that, in Tanzania the international migration component has been negligible. NBS (2012) showed that 99% of the enumerated population were Tanzanians. Majority of foreigners were refugees from neighbouring countries, it only affected the population in those areas at the boarders with those countries having civil war. Therefore, in this paper immigration component is not considered.

Exponential model

This model was proposed by the Englishman Thomas R. Malthus (Malthus 1798). The main assumption in this model is that the increase of population follows a geometric progression. Malthus did not take into account the fact that in any given environment, the growth of population may stop due to the density of population or competition of resources (Al-Eideh and Al-Omar 2019). He assumed that the rate of growth is proportional to the existing population (Cohen 1995).

Let the number of individuals at time t be denoted by $N(t)$. Malthusian law arises from the solution of the initial value Ordinary Differential equation.

$$\frac{dN(t)}{dt} = aN(t), \quad N(0) = N_0 \qquad (1)$$

where $N_0 > 0$ denotes the initial population size and $a = b - \mu$, which is the constant per capita growth rate of the population, that is, the average per person number of offspring b less the per person average number of deaths μ (Marsden et al. 2003). The assumption of a constant per capita growth rate leads to the solution of Equation (2).

$$N(t) = N_0 e^{at}, \qquad (2)$$

which predicts the population growth if $a > 0$, population decrease if $a < 0$, or no changes if $a = 0$.

The model may be impractical when the number of generations gets large enough for other factors to come into play.

Logistic model

Logistic model was developed by Belgian mathematician Pierre Verhulst in 1838 (Brauer and Castillo-Chavez 2001). He commented that the population growth depends on carrying capacity and the maximum rate of growth. He suggested the population growth to be limited. This means the rate of population growth may change depending on relationship between the initial population and carrying capacity. Pierre Verhulst modified Malthusian model by including the term

$$\frac{a - bN(t)}{a} \qquad (3)$$

where a and b are coefficients of the population. This term describes how far the population is from its limiting value. Substitute Equation (3) as a coefficient in Malthusian model (1) to get

$$\frac{dN(t)}{dt} = \frac{aN(t)(a - bN(t))}{a}. \qquad (4)$$

On simplifying Equation (4) we obtain the logistic differential equation which is written as

$$\frac{dN(t)}{dt} = aN(t)\left(1 - \frac{N(t)}{K}\right) \qquad (5)$$

where $K = \frac{a}{b}$.

In Equation (5), K is the carrying capacity, $N(t)$ is the function depending on t giving the population at time t, and a is constant of proportionality. Solving Equation (5) by separation of variables technique, we obtain

$$\int \frac{dN(t)}{N(t)\left(1 - \frac{N(t)}{K}\right)} = \int a dt \qquad (6)$$

To evaluate the left hand side we write

$$\frac{1}{N(1 - N/K)} = \frac{K}{N(K - N)} = \frac{1}{N} + \frac{1}{K - N} \qquad (7)$$

Hence

$$\int \frac{dN}{N} + \int \frac{dN}{K - N} = \int a dt$$

$$\ln|N| - \ln|K - N| = at + c$$

$$\ln\left|\frac{K - N}{N}\right| = -at - c$$

$$\left|\frac{K - N}{N}\right| = e^{-at-c}$$

$$\frac{K - N}{N} = Ae^{-at},$$

where $A = \pm\, e^{-c}$

Thus,

$$N = \frac{K}{1 + Ae^{-at}} \qquad (8)$$

where $A = \frac{K - N_0}{N_0}$

Hence,

$$N(t) = \frac{K}{1 + \left(\frac{K - N_0}{N_0}\right)e^{-at}} \qquad (9)$$

The next section presents the summary of results after running the two models using NBS data in Matlab software.

Summary of Results and Discussion

For exponential model, the first step is to find the growth rate a. By using equation (2) and the population of 1967 census as N_0 and of 2012 (last census) as $N(t)$ for a period $t = 45$. The growth rate of $a = 2.88\%$ is obtained. This growth rate is high according to African countries standards (NBS 2012).

For logistic model, the population of 1967 census which is initial population and population in 2012 as $N(t)$ is applied. The growth rate is obtained from exponential model which is 2.88%. By substituting all these in Equation 9, the carrying capacity is equal to $K = 2{,}976{,}857{,}550$.

Comparison with real data from five censuses

First the comparison of the model and real data from the five censuses is done.

The growth rate of 2.88% is used and the carrying capacity of $K = 2{,}976{,}857{,}550$ for logistic model. Figure 1 shows the trend of the population growth of the actual data from five censuses and the population projected by using exponential and logistic models.

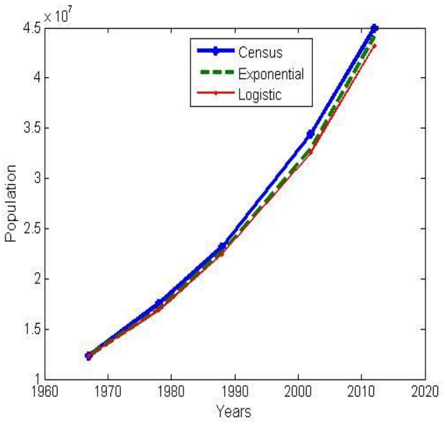

Figure 1: Comparison of models and five censuses data.

The exponential model projection performed better compared to the logistic. Its values are closer to real values obtained from five national censuses.

Comparison with NBS projections from 2013 to 2035

Here, the comparison between NBS projections and the two models from 2013 to 2035 is done. NBS projections show that Tanzania population in 2035 will be 89,204,781.

The exponential model gives a value of 87,538,767 and logistic model gives the population of 85,102,504. The values of exponential model are closer to the NBS projections compared to logistic model.

Figure 2 shows the comparison of population projections from 2013 to 2035 of both models with NBS projections including the population from 2012 census.

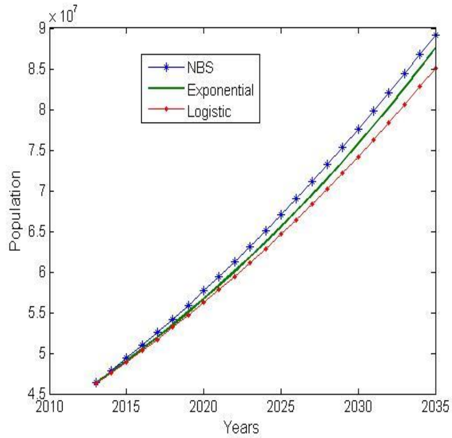

Figure 2: Projection from 2013 to 2035.

Projections from 2013 to 2050

According to the Unies (2017), Tanzania population is expected to reach 137 million by 2050. We project for a period from 2013 to 2050 by using both exponential and logistic models and compare.

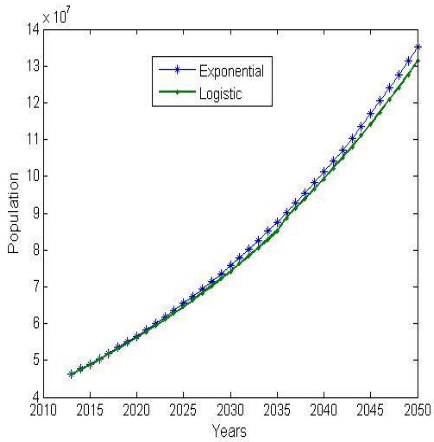

Figure 3: Projection from 2013 to 2050.

Figure 3 shows that by 2050 the population of Tanzania will be 135,244, 161 by exponential model which is closer to that projected by the United Nations. The logistic model projects the population of 131,261,794.

The two models give similar results and slightly deviate on higher levels indicating the different views as built from the two models. The exponential model does not take into consideration the slowdown effect associated with growing scarcity of resources with population. Logistic model on the other hand considers the slowdown effect as we approach the carrying capacity and hence the slight differences in growth at the end. Despite of the differences, the two models still project high growth and cause for alarm if the population does not match with economic growth.

Conclusion

In this paper, popular mathematical models for population projections, which are exponential and logistic, are used. They describe the dynamics of populations with overlapping generations which are best described using differential equations (Marsden et al. 2003). Numerical results have shown that mathematical model can be used to give better and less costly projections. Also mathematical models can be validated by comparing results with past actual data.

As shown in this paper, the population in Tanzania is growing fast compared to other countries. Madulu (2005) asserted that the increase in population may lead to expansion of some features that will reduce the carrying capacity of the land and the capability of the common property resources to meet the needs of the present and future populations. Therefore, the Government should take measures to reduce the effect of this growth including formulation of policies to protect environment and natural resources as stated in the national population policy. On the other hand, this can help to plan for public social services provision particularly education, water and health.

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Appendices

Table 3: Comparison between five censuses data and data projected models

Year	Census	Exp.	Logistic
1967	12313469	12313469	12313469
1978	17512610	16940271	16870412
1988	23174335	22639444	22450573
2002	34443603	33006143	32519392
2012	44928923	44110317	43200160

Table 4: Comparison of NBS, exponential and logistic projections

Year	NBS	Exp.	Logistic
2012	44928293	44928923	44928923
2013	46356279	46250938	46209894
2014	47831361	47611853	47526186
2015	49359408	49012812	48878701
2016	50941672	50454995	50268364
2017	52554628	51939612	51696117
2018	54199163	53467914	53162923
2019	55890747	55041186	54669764
2020	57637628	56660751	56217642
2021	59441988	58327970	57807578
2022	61280743	60044247	59440615
2023	63150477	61811025	61117813
2024	65070448	63629789	62840254
2025	67036280	65502070	64609040
2026	69054148	67429442	66425293
2027	71106934	69413526	68290155
2028	73197399	71455991	70204787
2029	75344775	73558554	72170371
2030	77537166	75722985	74188109
2031	79776176	77951103	76259221
2032	82057431	80244783	78384948
2033	84380103	82605953	80566548
2034	86766463	85036600	82805301
2035	89204781	87538767	85102504

Table 5: Comparison of exponential and logistic models projections from 2012-2050

Year	Exponential	Logistic	% Diff.
2012	44928923	44928923	0.0
2013	46250938	46209894	0.1
2014	47611853	47526186	0.2
2015	49012812	48878701	0.3
2016	50454995	50268364	0.4
2017	51939612	51696117	0.5
2018	53467914	53162923	0.6
2019	55041186	54669764	0.7
2020	56660751	56217642	0.8
2021	58327970	57807578	0.9
2022	60044247	59440615	1.0
2023	61811025	61117813	1.1
2024	63629789	62840254	1.2
2025	65502070	64609040	1.4
2026	67429442	66425293	1.5
2027	69413526	68290155	1.6
2028	71455991	70204787	1.8
2029	73558554	72170371	1.9
2030	75722985	74188109	2.0
2031	77951103	76259221	2.2
2032	80244783	78384948	2.3
2033	82605953	80566548	2.5
2034	85036600	82805301	2.6
2035	87538767	85102504	2.8
2036	90114560	88767165	1.5
2037	92766144	91298996	1.6
2038	95495750	93900692	1.7
2039	98305674	96574046	1.8
2040	101198279	99320889	1.9
2041	104175997	102143088	2.0
2042	107241333	105042553	2.1
2043	110396866	108021231	2.2
2044	113645249	111081107	2.3
2045	116989215	114224208	2.4
2046	120431575	117452600	2.5
2047	123975225	120768389	2.6
2048	127623146	124173720	2.7
2049	131378405	127670780	2.8
2050	135244161	131261794	2.9