Tanzania Journal of Science 49(1): 48-61, 2023
ISSN 0856-1761, e-ISSN 2507-7961

Simulation of Mathematical Model of Network Interference on Global System for Mobile Communication

Yidiat O. Aderinto* and Yusuf Musa
Mathematics Department, Faculty of Physical Science, P.M.B.1515, University of Ilorin, Ilorin, Nigeria.
Email addresses: aderinto@unilorin.edu.ng; muyuempire@gmail.com
**Corresponding author Email: aderinto@unilorin.edu.ng*
Received 22 Sep 2022, Revised 17 Feb 2023, Accepted 20 Feb 2023 Published Mar 2023
DOI: https://dx.doi.org/10.4314/tjs.v49i1.5

Abstract

Network interference is the incorporation of undesirable signals to desirable ones in an interconnected system. In this paper, the simulation of mathematical model of network interference with respect to global system of mobile communication is presented. The model was formulated and analyzed mathematically. Numerical simulation was carried out using real life data from Communication Towers Nigeria Limited, Northwest regional office Kaduna, Nigeria. The results obtained were very close to that of laboratory investigation.

Keywords: Mathematical Model, Network, Interference, Congestion, Mobile communication, Stability.

Introduction

The social, economic and political developments of the nation have improved positively without doubt due to the implementation of Global System for Mobile Communication (GSM) services. It also serves as income generator to the government (Pipikakis 2004, Adegoke et al. 2008, Popoola et al. 2009, Lawal et al. 2016, Adediran et al. 2017).

However, in the communication network, interference modifies signal in a disruptive manner resulting to abnormal quality of services (QoS) which affects majorly the accessibility, network coverage, stability of connection, quality of service integrity, and defeats the benefits of GSM telephony (NCC 2003, Adediran et al. 2017). Hence the needs for monitoring, analysis and improvement of quality of service is essential. So, many techniques have been proposed to minimize effects of interference in networks.

Many authors have worked on network interference with reference to evaluation and analysis of QoS of Mobile Cellular Networks, these include the following: Brehmer and Utschick (2010) worked on optimal management of interference in both multi-antenna and multi-cell systems. Adediran et al. (2017) studied the interference management techniques in cellular networks. Sumila and Miszkiewicz (2015) studied the problem of interference and the impact on the receivers. Alexander et al. (2013) employed drive test to study the QoS of communication networks using an ANFIS algorithm. Indigo et al. (2012) used network performance index to investigate and analyze QoS of Visafone network in Nigeria. Ugwoke et al. (2014) used a software drive test to examine QoS on mobile network in Nigeria. Nochiri et al. (2014) employed real time methodology as well as established KPIs to carry out a study on QoS of the Nigerian GSM.

However, little or less has been done on the mathematical model of network interference, hence this paper focused on characterizing and simulating mathematical model to investigate the effects of network interference on global system for mobile communication (especially MTN, Glo, 9mobile and Airtel services).

Materials and Methods

Model formulation

Interference, congestion in traffic and use of outdated equipment to certain number of subscribers using the network are the major inhibitions to the quality of services. Hence, the mathematical model of network interference is formulated and characterized in attempts to study the QoS provided by GSM operatives. The formulation in this paper is thus based mainly on interference and four (4) GSM networks compartmental models, namely MTN, Glo, 9Mobile and Airtel were considered. KPI features like call drop, call fade, call setup success (CSS) and cell data throughput were all investigated. The proportion of subscribers on a network (k*i*) is independent on the others while interference rate (β) can be carried to affect cell throughput. Call drop (μ) in either of the network is assumed varies but call fading (α) is the same because of combined interference. The following assumptions were imposed.

1. Call blocked is independent on one another.
2. Call drop varies depending on location, old age of radio/facilities at the base station and subscribers attempting to make call at particular time t.
3. Interference is assumed the same for all networks at time t but congestion is not except if the attempting to call single network and same SIM.
4. Only calls/network, which is less affected by interference has cell data throughput (γ).
5. Call Setup Success Rate (CSSR) is assumed to be homogeneous for all networks. It is also assumed that not all call setup successes enjoy cell data throughput. Removal rate due to interference is (μ + α) and total time expired before removal is $\frac{1}{(\mu+\alpha)}$.
6. It is assumed that some mobile devices pull network than another, depending on the device capacities.
7. CSSR is assumed $\beta(k_1M + k_2G + k_3N + k_4A)I$, the product of number of subscribers attempting and proportion of interference.
8. Subscribers are assumed either to exit cell data throughput voluntarily or they may be willing to terminate/pullout of the call before cell data throughput if poor connectivity/bad network is experienced.

The descriptions of the parameters and variables are presented in Table 1.

Flow diagram of MTN (M), GLO (M), 9Mobile (N) and Airtel (A) subscribers attempting to make calls at a particular time but being affected by the same Interference (I) while some subscribers were able to make call Successfully (S).

Where $T = M + G + N + A + I + S$ (Total number of subscribers attempted calls).

With the flow diagram in Figure 1, the model equation is obtained as:

$$\begin{aligned}
\mathrm{dM/dt} &= \Lambda_1 - \mu_1 \mathrm{M} - (\beta k_1 \mathrm{MI})/\mathrm{T} \\
\frac{dG}{dt} &= \Lambda_2 - \mu_2\mathrm{G} - \frac{\beta k_2 GI}{T} \\
\frac{dN}{dt} &= \Lambda_3 - \mu_3\mathrm{N} - \frac{\beta k_3 NI}{T} \\
\frac{dA}{dt} &= \Lambda_4 - \mu_4\mathrm{A} - \frac{\beta k_4 AI}{T} \\
\frac{dI}{dt} &= \beta(k_1M + k_2G + k_3N + k_4A)\frac{I}{T} - (\mu + \alpha + \gamma)\mathrm{I} \\
\frac{dS}{dt} &= \gamma I - \alpha_2\mathrm{S}
\end{aligned} \tag{1}$$

Table 1: Descriptions of parameters and variables (obtained from Communication Towers Nigeria Limited, Northwest Regional Office Kaduna)

Parameter / Variable	Description	Value
k_1	Success rate on MTN	1.0
k_2	Success rate on Glo	0.92
k_3	Success rate on 9 Mobile	0.79
k_4	Success rate on Airtel	1.0
β	Interference rate on the network	[0.01, 0.5]
μ_1	Proportion of call drop as a result of congestion on MTN	0.02
μ_2	Proportion of call drop as a result of congestion on Glo	0.01
μ_3	Proportion of call drop as a result of congestion on 9 Mobile	0.01
μ_4	Proportion of call drop as a result of congestion on Airtel	0.04
Λ_1	Subscribers to MTN	10280
Λ_2	Subscribers to Glo	4685
Λ_3	Subscribers to 9 Mobile	2054
Λ_4	Subscribers to Airtel	7378
α	Call fading proportion due to combination of interference and congestion on call setup	0.01
α_2	Voluntary removal	0.91
γ	Cell data throughput rate	0.92
μ	Call drop due to poor service	0.05
M_0	Initial subscribers to MTN	1500
G_0	Initial subscribers to Glo	1100
N_0	Initial subscribers to 9 Mobile	200
A_0	Initial subscribers to Airtel	1300
I_0	Initial subscribers susceptible to interference and congestion	500
S	Initial successful subscribers	200

Flow diagram

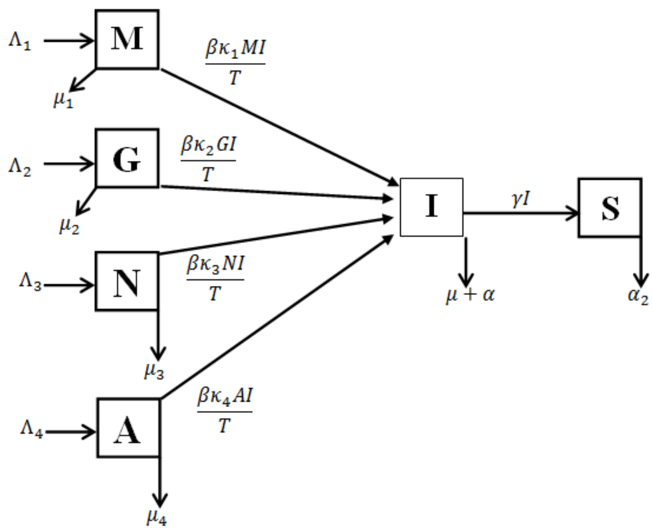

Figure 1: Model flow diagram.

Existence and uniqueness of solution to the model

The existence and uniqueness of equation (1) shall be investigated using Lemma 1

Lemma 1: (Madhu and Harish 2014)

A function, $f : \mathbb{R}^n \rightarrow \mathbb{R}^n$ is called locally Lipschitz at $x_0 \in \mathbb{R}^n$ if there exist a neighborhood B(x_0; ϵ) with $\epsilon > 0$: $\left|\left|f(x_1 - x_2)\right|\right| \leq L\left|\left|x_1 - x_2\right|\right|$, L > 0, $x1$ and $x_2 \in B(x_0, \epsilon)$. $B(x_0, \epsilon) \coloneqq \left\{x \in \mathbb{R}^n \mid \left|\left|x - x_0\right|\right| < \epsilon\right\}$.

Where B(x_0, ϵ) $\subset R^n$ is an open ball around x_0 of radius ϵ, L is called a Lipschitz constant.

Theorem 1

Suppose $F_i(t, x), x(t_0) = x_0, i = 1, 2, \dots, n$ exists. Let $F_i(t, M, G, N, A, I, S), i(1)6$ satisfies the Lipschitz condition, then the solution to equation (1) exists and unique.

Proof: Using Lemma 1

Rewrite (1) as

$$
\begin{aligned}
F_1(t, M) &= \Lambda_1 - \mu_1 M - \frac{\beta k_1 M I}{T} \\
F_2(t, G) &= \Lambda_2 - \mu_2 G - \frac{\beta k_2 G I}{T} \\
F_3(t, N) &= \Lambda_3 - \mu_3 N - \frac{\beta k_3 N I}{T} \\
F_4(t, A) &= \Lambda_4 - \mu_4 A - \frac{\beta k_4 A I}{T} \\
F_5(t, c) &= \beta(k_1 M + k_2 G + k_3 N + k_4 A)\frac{I}{T} - (\mu + \alpha + \gamma) I \\
F_6(t, S) &= \gamma I - \alpha_2 S
\end{aligned}
\qquad (2)
$$

Where $c = M, G, N, A$; define (2) in the form $X_i' = F_i(t, x), x(t_0) = x_0, i(1)6$, then, $|F_i(t, x_2) - F_i(t, x_1)| \leq L|x_2 - x_1|$

For F_1

$$F_1(t, M_2) - F_1(t, M_1) = \Lambda_1 - \mu_1 M_2 - \frac{\beta k_1 M_2 I}{T} - \left(\Lambda_1 - \mu_1 M_1 - \frac{\beta k_1 M_1 I}{T}\right)$$

$$F_1(t, M_2) - F_1(t, M_1) = -\left(\mu_1 + \frac{\beta k_1 I}{T}\right)(M_2 - M_1)$$

$$|F_1(t, M_2) - F_1(t, M_1)| \leq \left|\mu_1 + \frac{\beta k_1 I}{T}\right| |M_2 - M_1|$$

For F_2

$$F_1(t, G_2) - F_1(t, G_1) = \Lambda_1 - \mu_1 G_2 - \frac{\beta k_1 G_2 I}{T} - \left(\Lambda_1 - \mu_1 G_1 - \frac{\beta k_1 G_1 I}{T}\right)$$

$$F_1(t, G_2) - F_1(t, G_1) = -\left(\mu_1 + \frac{\beta k_1 I}{T}\right)(G_2 - G_1)$$

$$|F_2(t, G_2) - F_1(t, G_1)| \leq \left|\mu_2 + \frac{\beta k_2 I}{T}\right| |G_2 - G_1|$$

For F_3

$$F_1(t, N_2) - F_1(t, N_1) = \Lambda_1 - \mu_1 N_2 - \frac{\beta k_1 N_2 I}{T} - \left(\Lambda_1 - \mu_1 N_1 - \frac{\beta k_1 N_1 I}{T}\right)$$

$$F_1(t, N_2) - F_1(t, N_1) = -\left(\mu_1 + \frac{\beta k_1 I}{T}\right)(N_2 - N_1)$$

$$|F_3(t, N_2) - F_3(t, N_1)| \leq \left|\mu_3 + \frac{\beta k_3 I}{T}\right| |N_2 - N_1|$$

Similarly for F_4 to F_6.

Since F_1 to F_6 satisfies the Lipschitz inequality, hence, the solution to (1) exists and unique.

Theorem 2

Let D denote the region bounded by $0 \leq \mathbb{R} < \infty$. Then $\frac{dF_i\,(t,M,G,N,A,I,S)}{dt}$, $i = 1, \dots, 6$ is continuous in the region D, if $\left|\frac{\partial F_i\,(t,M,G,N,A,I,S)}{\partial t}\right| < \infty$.

Proof: From equation (2)

For F_1

$$\left|\frac{\partial F_1}{\partial M}\right| = \left|\mu_1 + \frac{\beta k_1 I}{T}\right| < \infty$$
$$\left|\frac{\partial F_1}{\partial G}\right| = 0 < \infty$$
$$\left|\frac{\partial F_1}{\partial N}\right| = 0 < \infty \qquad (3)$$
$$\left|\frac{\partial F_1}{\partial I}\right| = \left|\frac{\beta k_1 M}{T}\right| < \infty$$
$$\left|\frac{\partial F_1}{\partial S}\right| = 0 < \infty$$

For F_2

$$\left|\frac{\partial F_2}{\partial M}\right| = 0 < \infty$$
$$\left|\frac{\partial F_2}{\partial G}\right| = \left|\mu_2 + \frac{\beta k_2 I}{T}\right| < \infty$$
$$\left|\frac{\partial F_2}{\partial N}\right| = 0 < \infty$$
$$\left|\frac{\partial F_2}{\partial A}\right| = 0 < \infty \qquad (4)$$
$$\left|\frac{\partial F_2}{\partial I}\right| = \left|\frac{\beta k_2 G}{T}\right| < \infty$$
$$\left|\frac{\partial F_2}{\partial S}\right| = 0 < \infty$$

Similar results hold for F_3 to F_6.

Since $\left|\frac{\partial F_i\,(t,M,G,N,A,I,S)}{\partial t}\right| < \infty$; for all $i = 1, \dots, 6$, hence (1) is continuous in the same region.

Equilibrium and stability analysis

Employing the definition of locally stable and locally asymptotically stable, the next theorems will be proved.

Theorem 3

Suppose model (1) is at equilibrium, then there exist E = (t; x) and $E^* = (t, x^*)$ where $x = M, G, N, A, I, S$ and $x^* = M^*, G^*, N^*, A^*, I^*, S^*$ that satisfying the equilibrium states.

Proof: Rewriting (1) in equilibrium form as:

$$\begin{aligned}
&\Lambda_1 - \mu_1 M - \frac{\beta k_1 MI}{T} = 0\\
&\Lambda_2 - \mu_2 G - \frac{\beta k_2 GI}{T} = 0\\
&\Lambda_3 - \mu_3 N - \frac{\beta k_3 NI}{T} = 0\\
&\Lambda_4 - \mu_4 A - \frac{\beta k_4 AI}{T} = 0\\
&\beta(k_1 M + k_2 G + k_3 N + k_4 A)\frac{I}{T} - (\mu + \alpha + \gamma)I = 0\\
&\gamma I - \alpha_2 S = 0
\end{aligned} \qquad (5)$$

Model (1) exhibits two equilibrium states; Interference Free Steady State and Persistence Steady State.
Case 1: For $I = 0$

$$E = \left\{(t, M, G, N, A, I, S): \frac{\Lambda_1}{\mu_1}, \frac{\Lambda_2}{\mu_2}, \frac{\Lambda_3}{\mu_3}, \frac{\Lambda_4}{\mu_4}, 0,0\right\}$$

Case 2: For $I \neq 0$

$$E^* = \left\{(t^*, M^*, G^*, N^*, A^*, I^*, S^*): \frac{T_{\Lambda_1}}{T\mu_1 + \beta k_1 I}, \frac{T\Lambda_2}{T\mu_2 + \beta k_2 I}, \frac{T\Lambda_3}{T\mu_3 + \beta k_3 I}, \frac{T\Lambda_1}{T\mu_4 + \beta k_4 I}, \frac{\alpha_2 S}{\gamma}, \frac{\gamma I}{\alpha_2}\right\}.$$

The concept of Jacobian matrix will be employed on equation (1) thus:

$$J = \begin{pmatrix}
-\mu_1 - \frac{\beta k_1 I}{T} & 0 & 0 & 0 & -\frac{\beta k_1 M}{T} & 0\\
0 & -\mu_2 - \frac{\beta k_2 I}{T} & 0 & 0 & -\frac{\beta k_2 G}{T} & 0\\
0 & 0 & -\mu_3 - \frac{\beta k_3 I}{T} & 0 & -\frac{\beta k_3 N}{T} & 0\\
0 & 0 & 0 & -\mu_4 - \frac{\beta k_4 I}{T} & -\frac{\beta k_4 A}{T} & 0\\
\frac{\beta k_1 I}{T} & \frac{\beta k_2 I}{T} & \frac{\beta k_3 I}{T} & \frac{\beta k_3 I}{T} & \omega & 0\\
0 & 0 & 0 & 0 & \gamma & -\alpha_2
\end{pmatrix} \qquad (6)$$

Theorem 4
Let (6) be the Jacobian matrix for model (1), then it will be locally asymptotically stable for all λ_i, $i(1)6$, if the eigenvalues have strictly negative roots.

Proof:
Case 1: Interference Free State; $|J - \lambda I| = 0$
Obtain the determinant of (6) above as

$$|J-\lambda I| = \begin{vmatrix} -(\mu_1+\lambda) & 0 & 0 & 0 & -\frac{\beta k_1 M}{T} & 0 \\ 0 & -(\mu_2+\lambda) & 0 & 0 & -\frac{\beta k_2 G}{T} & 0 \\ 0 & 0 & -(\mu_3+\lambda) & 0 & -\frac{\beta k_3 N}{T} & 0 \\ 0 & 0 & 0 & -(\mu_4+\lambda) & -\frac{\beta k_4 A}{T} & 0 \\ 0 & 0 & 0 & 0 & \omega-\lambda & 0 \\ 0 & 0 & 0 & 0 & \gamma & -(\alpha_2+\lambda) \end{vmatrix}$$

Which implies:

$$|J-\lambda I| = -(\mu_1+\lambda)\big(-(\mu_2+\lambda)\big)\big\{\big(-(\mu_3+\lambda)\big)\big(-(\mu_4+\lambda)\big)[-(\omega-\lambda)(\alpha_2+\lambda)]\big\} = 0$$

$$= (\mu_1+\lambda)(\mu_2+\lambda)(\mu_3+\lambda)(\mu_4+\lambda)(\omega-\lambda)(\alpha_2+\lambda) = 0$$

Thus, the eigenvalues are:

$$\lambda_1 = -\mu_1,\ \lambda_2 = -\mu_2,\ \lambda_3 = -\mu_3,\ \lambda_4 = -\mu_4,\ \lambda_5 = -\mu_5, \text{ and } \lambda_6 = \omega$$

Case 2: Interference Persistence State; $|J^* - \lambda I^*| = 0$
From (6)

$$|J^*-\lambda I^*| = \begin{vmatrix} B & 0 & 0 & 0 & -\frac{\beta k_1 M^*}{T} & 0 \\ 0 & F & 0 & 0 & -\frac{\beta k_2 G^*}{T} & 0 \\ 0 & 0 & G & 0 & -\frac{\beta k_3 N^*}{T} & 0 \\ 0 & 0 & 0 & H & 0 & -\frac{\beta k_4 A^*}{T} \\ \frac{\beta k_1 I}{T} & \frac{\beta k_2 I}{T} & \frac{\beta k_3 I}{T} & \frac{\beta k_4 I}{T} & L & 0 \\ 0 & 0 & 0 & 0 & \gamma & Q \end{vmatrix} \quad (7)$$

Where

$$B = -\left(\mu_1 + \frac{\beta k_1 I}{T} + \lambda^*\right)$$

$$F = -\left(\mu_2 + \frac{\beta k_2 I}{T} + \lambda^*\right)$$

$$G = -\left(\mu_3 + \frac{\beta k_3 I}{T} + \lambda^*\right)$$

$$H = -\left(\mu_4 + \frac{\beta k_4 I}{T} + \lambda^*\right)$$

$$L = (\omega^* - \lambda^*)$$

$$Q = -(\alpha_2 + \lambda^*)$$

and $\omega^* = \frac{\beta}{T}(k_1 M^* + k_2 G^* + k_3 N^* + k_4 A^*) - (\mu + \alpha + \gamma)$

Thus:

$$|J^* - \lambda I^*| = -\left(\mu_1 + \frac{\beta k_1 I}{T} + \lambda^*\right)\left(-\left(\mu_2 + \frac{\beta k_2 I}{T} + \lambda^*\right)\right)\left\{-\left(\mu_3 + \frac{\beta k_3 I}{T} + \lambda^*\right)(-(\mu_4 + \frac{\beta k_4 I}{T} + \lambda^*)\right)\left[-(\omega - \lambda^*)(-(\alpha_2 + \lambda^*))\right]\right\} = 0$$

$$= \left(\mu_1 + \frac{\beta k_1 I}{T} + \lambda^*\right)\left(\mu_2 + \frac{\beta k_2 I}{T} + \lambda^*\right)\left(\mu_3 + \frac{\beta k_3 I}{T} + \lambda^*\right)\left(\mu_4 + \frac{\beta k_4 I}{T} + \lambda^*\right)(\omega - \lambda^*)(\alpha_2 + \lambda^*) = 0$$

The eigenvalues are:

$$\lambda_1^* = -\left(\mu_1 + \frac{\beta k_1 I}{T}\right),$$

$$\lambda_2^* = \frac{\beta}{T}(k_1 M^* + k_2 G^* + k_3 N^* + k_4 A^*) < (\mu^* + \alpha^* + \gamma^*) = -\left(\mu_2 + \frac{\beta k_2 I}{T}\right),$$

$$\lambda_3^* = -\left(\mu_3 + \frac{\beta k_3 I}{T}\right), \lambda_4^* = -\left(\mu_4 + \frac{\beta k_4 I}{T}\right), \lambda_5^* = -\alpha_2 \; and \; \lambda_6^* = \omega^*$$

The eigenvalues for Cases 1 and 2 returned to negative but λ_6 and λ_6^* require further analysis, the case where $\frac{\beta}{T}(k_1 M + k_2 G + k_3 N + k_4 A) < (\mu + \alpha + \gamma)$
and $\frac{\beta}{T}(k_1 M^* + k_2 G^* + k_3 N^* + k_4 A^*) < (\mu + \alpha + \gamma)$ before conclusion can be made on the stability analysis.

Numerical application

The deterministic model investigating the effect of network interference on the QoS provided by mobile network operators was solved using Classical Fourth Order Runge-Kutta (Order-4) method due to its effectiveness and accuracy over other lower order Runge-Kutta methods (Jain 1984, Gowri et al. 2017).

The data were obtained from DT conducted by Communication Towers Nigeria Limited, Northwest Regional Office Kaduna and the study is limited to Kaduna metropolis especially Barnawa Area, Kawo Area and Constitution Road Area of Kaduna State. Maple-18 software was used to run the simulations and the results are shown in Tables 2 and 3, the behaviours are graphically represented in Figures 2 to 8.

Table 2: Values obtained from KPI for Interference Free State

β	μ_1	μ_2	μ_3	μ_4	k_1	k_2	k_3	k_4	Λ_1	Λ_2	Λ_3	Λ_4	λ_6	$Remark$
0.01	0.02	0.01	0.01	0.04	1.00	0.92	1.00	0.79	4550	2074	909	3267	-0.979991	Stable
0.03	0.02	0.01	0.01	0.04	0.99	0.91	0.98	0.78	4550	2074	909	3267	-0.979998	Stable
0.05	0.02	0.01	0.01	0.04	0.97	0.90	0.95	0.77	4550	2074	909	3267	-0.979995	Stable
0.18	0.02	0.01	0.04	0.03	0.75	1.00	0.89	1.00	3118	1421	623	2239	-0.929985	Stable
0.20	0.02	0.01	0.04	0.03	0.73	0.97	0.86	0.98	3118	1421	623	2239	-0.929983	Stable
0.23	0.02	0.01	0.04	0.03	0.70	0.96	0.83	0.97	3118	1421	623	2239	-0.929984	Stable
0.31	0.01	0.02	0.02	0.01	0.98	0.99	0.99	0.98	2612	1190	522	1876	-1.059958	Stable
0.40	0.01	0.02	0.02	0.01	0.71	0.77	0.73	0.71	2612	1190	522	1876	-1.019955	Stable
0.48	0.01	0.02	0.02	0.01	0.60	0.59	0.63	0.58	2612	1190	522	1876	-1.019953	Stable

Table 3: Values obtained from KPI Interference Persistence State

β	μ_1	μ_2	μ_3	μ_4	k_1	k_2	k_3	k_4	Λ_1	Λ_2	Λ_3	Λ_4	λ_6^*	$Remark$
0.01	0.02	0.01	0.01	0.04	1.00	0.92	1.00	0.79	4550	2074	909	3267	-0.979994	Stable
0.03	0.02	0.01	0.01	0.04	0.99	0.91	0.98	0.78	4550	2074	909	3267	-0.989997	Stable
0.05	0.02	0.01	0.01	0.04	0.97	0.90	0.95	0.77	4550	2074	909	3267	-0.979995	Stable
0.18	0.02	0.01	0.04	0.03	0.75	1.00	0.89	1.00	3118	1421	623	2239	-0.949986	Stable
0.20	0.02	0.01	0.04	0.03	0.73	0.97	0.86	0.98	3118	1421	623	2239	-0.939985	Stable
0.23	0.02	0.01	0.04	0.03	0.70	0.96	0.83	0.97	3118	1421	623	2239	-0.929982	Stable
0.31	0.01	0.02	0.02	0.01	0.98	0.99	0.99	0.98	2612	1190	522	1876	-1.069958	Stable
0.40	0.01	0.02	0.02	0.01	0.71	0.77	0.73	0.71	2612	1190	522	1876	-1.039955	Stable
0.48	0.01	0.02	0.02	0.01	0.60	0.59	0.63	0.58	2612	1190	522	1876	-1.019952	Stable

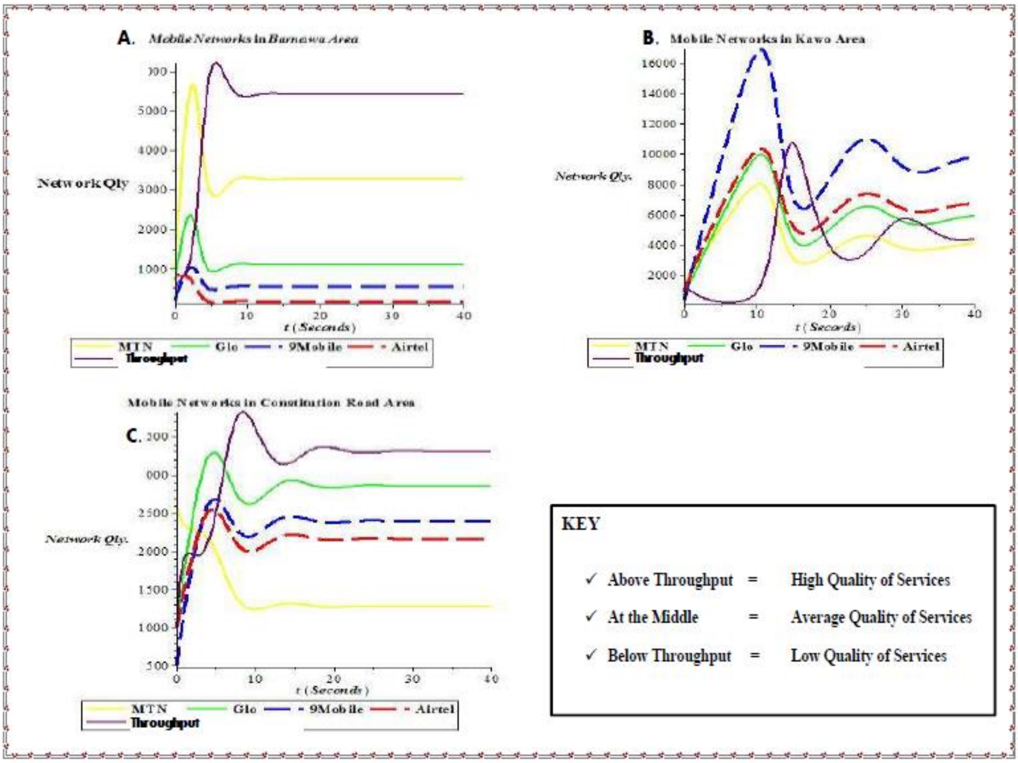

Figure 2: A-Behaviour of mobile networks in Barnawa Area; B-Behaviour of mobile networks in Kawo Area; C-Behaviour of mobile networks in Constitution Road Area.

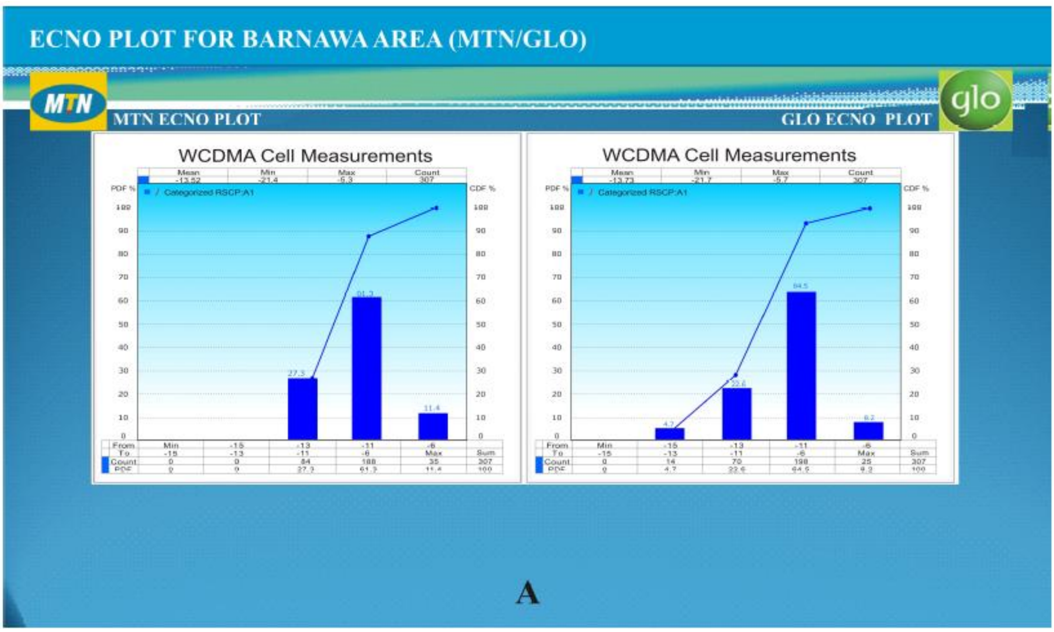

Figure 3: Graphical interpretation of DT for MTN and Glo at Barnawa Area.

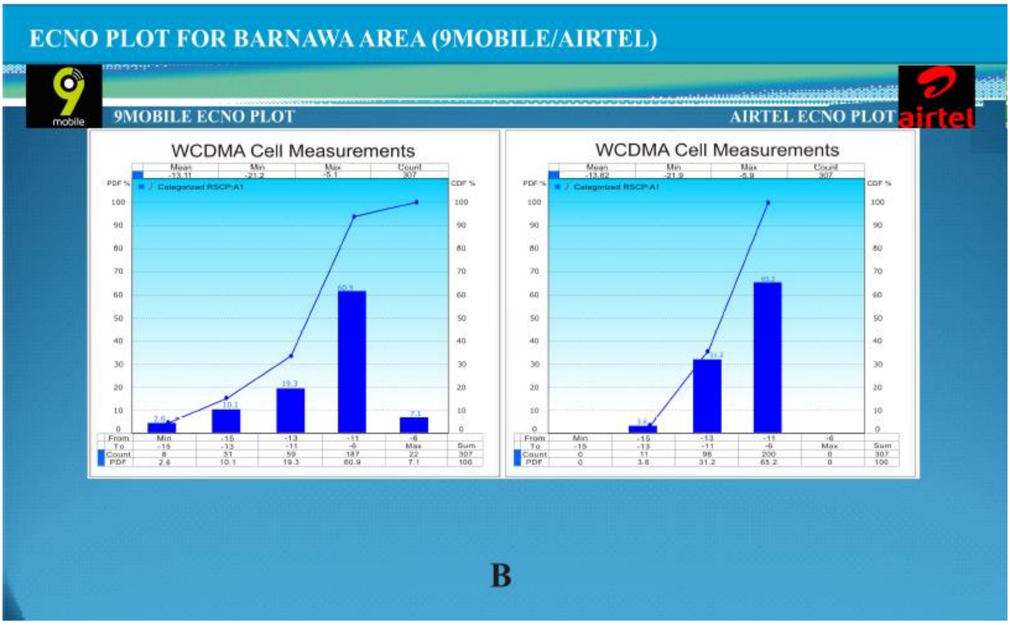

Figure 4: Interpretation of DT for 9Mobile and Airtel at Barnawa Area.

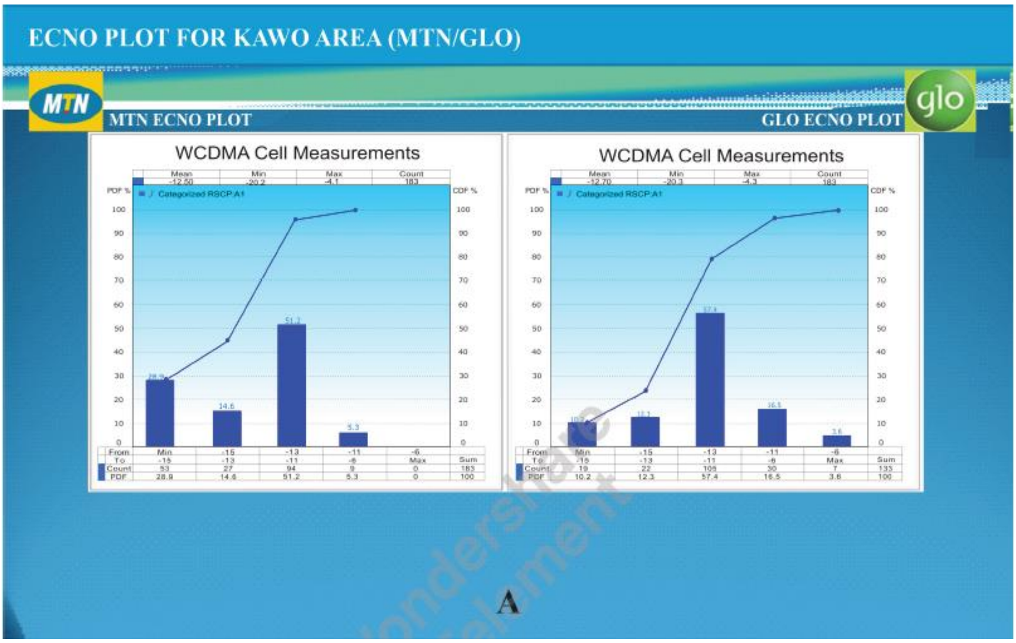

Figure 5: Interprets DT for MTN and Glo at Kawo Area.

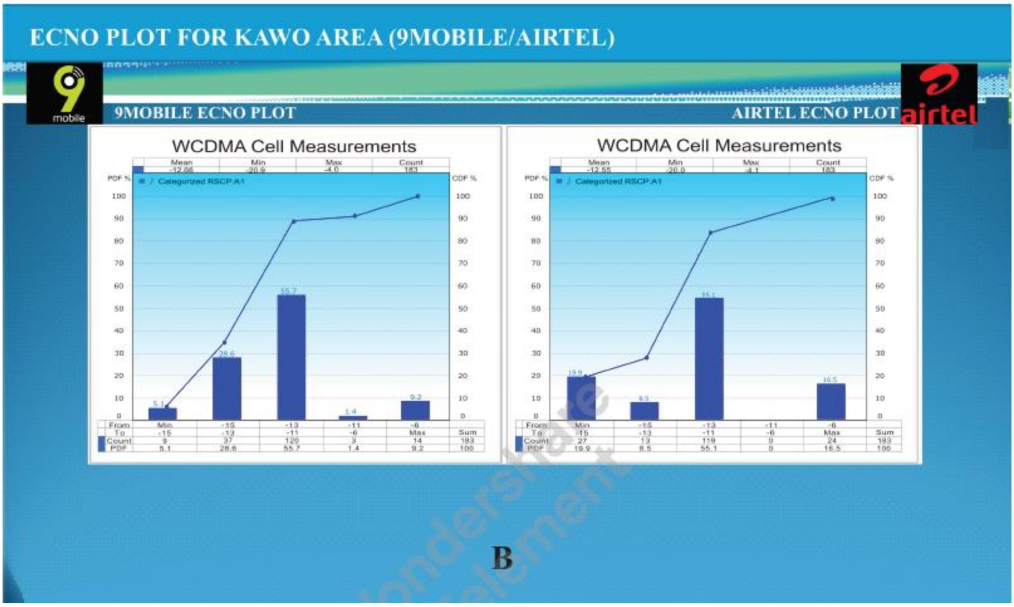

Figure 6: Interpretation for 9Mobile and Airtel at Kawo Area.

Figure 7: Reveals the interpretation of DT for MTN and Glo at Constitution Road Area.

Figure 8: Shows the interpretation of DT for 9Mobile and Airtel at Constitution Road Area.

Discussion of Results

The results obtained are discussed as follows. From Tables 2 and 3, λ_6 and λ_6^* were negative which implies that model (1) is stable everywhere for both low and high interference rate, thus the system is locally asymptotically stable.

Numerical simulation results presented in Figure 2 showed that;

At Barnawa area, interference rate was β = 0.31, while success rates were $k_1 = 0.73$, $k_2 = 0.97$, $k_3 = 0.86$, and $k_4 = 0.98$, which revealed high network quality for all the networks. At Kawo area, it also showed that interference rate of β = 0.48, while success rates were $k_1 = 0.60$, $k_2 = 0.59$, $k_3 = 0.63$, and $k_4 = 0.58$, thus average network quality for the four mobile networks. At Constitution road area, it was observed that interference rate was β = 0.20, while success rates were $k_1 = 0.98$, $k_2 = 0.99$, $k_3 = 0.99$, and $k_4 = 0.98$, which revealed high network quality for the four networks.

DT (conducted by Communication Towers Nigeria Limited) investigating the quality of network (i.e. ECNO) in the three areas is presented in graphical forms labelled Figures 3 to 8 and analyzed as follows: In Banawa Area, MTN (61.3%), Glo (64.5%), 9Mobile (60.9%) and Airtel (65.2%) showing that there is very good network quality for all four mobile networks. In Kawo Area, MTN (51.2%), Glo (57.4%), 9Mobile (55.7%), Airtel (55.1%) revealing good network quality for all the four networks. In Constitution Road Area, MTN (67.4%), Glo (62.7%), 9Mobile (69.3%) and Airtel (64.5%) signifying very good network quality for all the four mobile networks.

Conclusion

Mathematical models of interference with respect to the global system of mobile communication were presented. The models were analyzed, existence and uniqueness of solution to the models were investigated among others. Numerical implications and simulations were also carried out using Runge Kutta fourth order method via Mapple 18 software. The results obtained showed that interference has great influence on the QoS provided by network operators in all the three areas investigated because the high the interference, the low the quality of network and vice versa. The results of the laboratory investigation are close to that of the DT when compared. More so, it was established that the areas with lower population/subscribers had better network quality than areas with high population.

Conflict of Interests: Authors declare no conflict of interest regarding this work.

References

Adediran YA, Lasisi H and Okedere OB 2017 Interference management techniques in cellular networks: A review. *Cogent Eng.* 4(1): 1-10.

Adegoke AS, Babalola IT and Balogun WA 2008 Performance evaluation of GSM mobile system in Nigeria. *Pacific J. Sci. Technol.* 9(2): 436-441.

Alexander NN, Edeko FO and Orakwe SI 2013 Evaluation and optimization of quality of service (Qos) of mobile cellular networks in Nigeria. *Int. J. Inf. Commun. Technol. Res.* 3(9): 277-282.

Brehmer J and Utschick W 2010 Interference management in multi-antenna, multi-cell systems. *Proceedings of the International Zurich Seminar on Communications*. 134-137.

Communication Towers Nigeria Limited, Northwest Regional Office Kaduna.

Gowri P, Priyadharsini S and Maheswari T 2017 A case study on Runge-Kutta 4th order differential equation and its application. *Imper. J. Interdiscipl. Res.* 3(2): 134-139.

Indigo VE, Biebuma JJ and Tom DB 2012 Real-time assessments of QoS of mobile cellular networks in Nigeria. *Int. J. Eng. Invent.* 1(6): 64-68.

Jain MK 1984 Numerical Solution of Differential Equations. 2nd edition Wiley Eastern Limited. New Delhi, Calcutta

Lawal BY, Ukhurebor KE, Adekoya MA and Aigbe EE 2016 Quality of service and performance analysis of a GSM network in Eagle Square, Abuja and its environs, Nigeria. *Int. J. Sci. Eng. Res.* 7(8): 1-7

Madhu NB and Harish KP 2014 Nonlinear Dynamical Systems. Department of Electrical Engineering, IIT Bombay, India.

NCC (Nigeria Communication Commission) 2003. https://www.ncc.gov.ng

Nochiri IU, Osuagwu CC and Okafor KC 2014 Empirical analysis on the GSM network KPIs using real time methodology for a novel network integration. *Prog. Sci. Eng. Res. J.* 2(13): 092-107.

Pipikakis M 2004 Evaluating and improving the quality of service of second-generation cellular systems. *Bechtel Telecommun. Techn. J.* 2(2): 1-8.

Popoola JJ, Megbowon IO and Adeloye VSA 2009 Performance evaluation and improvement on quality of service of global system for mobile communications in Nigeria. *Comput. Science, Business, J. Inf. Technol. Impact* 9(2): 91-106

Sumila M and Miszkiewicz A 2015 Analysis of the problem of interference of the public network operators to GSM-R. *TST 2015, Inter Conference on Transport Systems Telematics* 531: 253-263

Ugwoke FN, Okafor KC, Onwusuru IM and Udeze CC 2014 Using software engineering approach in mitigating QoS challenges in mobile communication networks in Nigeria. *Comput. Inf. Syst. Dev. Informat. Allied Res. J.* 5(1): 131-148.