Tanzania Journal of Science 47(2): 432-441, 2021
ISSN 0856-1761, e-ISSN 2507-7961

Thermodynamic Analysis of a Variable Viscosity Reactive Hydromagnetic Couette Flow within Parallel Plates

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Received 29 Sep 2020, Revised 9 Mar 2021, Accepted 21 Apr 2021, Published May 2021
DOI: https://dx.doi.org/10.4314/tjs.v47i2.3

Abstract

This investigation is to consider the impact of a temperature-dependent variable viscosity of a reactive hydromagnetic Couette fluid flowing within parallel plates. The variable property of the fluid viscosity is thought to be an exponential relation of temperature under the impact of magnetic strength. The differential equations controlling the smooth movement of fluid and energy transfer are modeled and solved by using the series solution of modified Adomian decomposition technique (mADM). The outcomes are shown in tables and graphs for different estimations of thermophysical properties present in the flow regime together with the rate of entropy generation and irreversibility distribution outcome.

Keywords: Reactive fluids, Couette Flow, variable viscosity, hydromagnetic and modified Adomian decomposition method (mADM).

Introduction

The uses of non-Newtonian fluids in current innovations in industries have been tremendous and attractive to researchers because of their essentialness in the current day industrial and engineering procedures. Many problems concerning the fluid flow with thermophysical properties which sometimes are assumed to be constant, some of these properties may alter fluid temperature, most importantly, the fluid viscosity. As discussed in Hassan (2019), the prediction of heat transfer rate and flow speed completely, it is critical to consider the variation of viscosity with respect to temperature.

However, the study of Couette flow is remarkably necessary because of its industrial applications in numerous flows occurring in many procedures that have been modeled with diverse heat transfer circumstances. Therefore, such complex fluid flow has been comprehensively investigated by many researchers for different limit conditions and other effects, for instance, the investigation of the entropy generation in an unsteady MHD Couette flow of a viscous incompressible electrically conducting fluid with respect to magnetic field strength in a circular system conducted in Das et al. (2016a). In addition to that, Das et al. (2016b) further examined the compound impacts of spinning and the magnetic field on the transient MHD Couette flow of a reactive fluid in a rotating system. To buttress the significant study on Couette flow, Makinde et al. (2015) employed the first two laws of thermodynamics to examine the flow and thermal dissipation in MHD variable viscosity flow of a Couette flow in a spinning system. Other investigations on Couette flow and other physical properties are extensively presented in Makinde (2014), Theuri and Makinde (2014), and Vyas and Ranjan (2015).

Notably, the investigation of Hayat et al. (2016) on temperature dependent thermal conductivity in a stretched surface with variable thickness in stagnation flow highlighted the impact of chemical reaction between fluid and foreign mass.

Moreover, to prevent a danger or risk in a system that controls fluid behavior, it is imperative to take note of fluid flow and heat transfer conditions, particularly the fluid viscosity. It is well known that viscous heating is generated due to surface interaction between the fluid and the walls together with high shear rates that lead to high temperature; in actual fact, viscosity is the most delicate fluid property that causes change in temperature as supported and complimented in Makinde and Maserumule (2008). Hence, it is important to note that the diverse means of fluid viscosity due to change in temperature may remarkably affect the flow properties together with efficiency of lubrication in industrial tools. Many authors have investigated various models of variable viscosity of fluid flow characteristics, for instance, Eegunjobi et al. (2015) examined the compound effects of unsteady and temperature-dependent viscosity on the rate of entropy distribution in a generalized Couette flow. Additionally, Makinde (2008a) considered the consequences of variable viscosity fluid through parallel channels with convective cooling the walls with the temperature-dependent viscosity as an exponential function of temperature. In addition to that, Hassan (2019) examined the impact of heat source without magnetic strength influence on a variable viscosity reactive Couette fluid flow. Further studies involving properties of variable viscosity could be found (Salem 2007, Mukhopadhyay 2009, Kobo and Makinde 2010, Hassan and Gbadeyan 2013, Makinde et al. 2016a, Makinde et al. 2016b, Salawu and Oke 2018, Wang et al. 2020).

From application point of view, the present study is to determine the impact of variable viscosity on a reactive hydromagnetic Couette fluid between parallel moving upper and static lower plate with an exponential temperature-dependent viscosity. The impact of electromagnetic influence on any fluid flow is highly noticeable on a flow of viscous materials as extensively discussed in Salem (2007), Hassan et al. (2017), and Muhammad et al. (2020). The model problem is obtained and solved analytically using a series solution of modified Adomian decomposition method (mADM). The choice of this technique is based on the rapid convergence with less iteration and has been shown in literature (Adomian 1994, Wazwaz 1999, Wazwaz and El-Sayed 2001) to be effective and accurate. The expressions from the outcomes of velocity and heat transfer are employed to determine the rate of entropy distribution, irreversibility distribution ration and Bejan number. Pertinent outcomes are shown in tables and graphs to demonstrate the remarkable impact of magnetic field strength and other thermophysical properties in the flow stream.

Problem Formulations

We consider a steady flow of an incompressible, reactive, viscous and hydromagnetic fluid with moving upper plate and stationary lower plate as shown in the configuration in Figure 1. The fluid viscosity is assumed to be an exponential function of temperature between two parallel plates of width, *H* and length, *L*. The upper plate is driven with uniform speed, *U*, while the lower plate is kept unmoved.

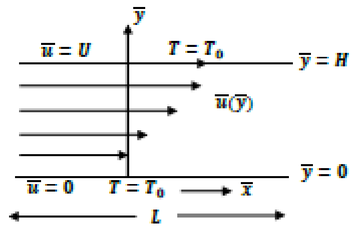

Figure 1: Geometry of the problem

The variable property of the fluid viscosity is taken to be an exponential function of temperature as used in Makinde (2008b), Makinde and Maserumule (2008), Gbadeyan and Hassan (2012), Eegunjobi et al. (2015), Makinde et al. (2015) and Hassan (2019) as:

$$\bar{\mu} = \mu_0 e^{-\frac{E}{R\bar{T}}} \qquad (1)$$

where μ_0, *E*, *R* and *T*, respectively stand for the fluid reference dynamic viscosity at a very large temperature (i.e. as $T \to \infty$), the activation energy, the universal gas constant and the absolute temperature.

The equations controlling the fluid flow in dimensional form without the reactant consumption are given as described in Makinde and Maserumule (2008), Kobo and Makinde (2010), and Hassan (2019) as:

$$\frac{d}{d\bar{y}}\left(\bar{u}\frac{d\bar{u}}{d\bar{y}}\right) - \sigma\frac{\beta_0^2}{\rho}\bar{u} = 0 \qquad (2)$$

$$k\frac{d^2\bar{T}}{d\bar{y}^2} + \bar{\mu}\left(\frac{d\bar{u}}{d\bar{y}}\right)^2 + QC_0Ae^{-\frac{E}{R\bar{T}}} + \sigma\frac{\beta_0^2}{\rho}\bar{u}^2 = 0 \qquad (3)$$

such that the initial boundary situations are given as:

$$\bar{u}(0) = \bar{T}(0) = 0 \ , \quad \bar{u}(1) = 1 \quad \text{and} \quad \bar{T}(1) = 0 \qquad (4)$$

We thereby introduce the dimensionless variables and parameters as follows:

$$y = \frac{\bar{y}}{L}, \quad x = \frac{\bar{x}}{L}, \ u = \frac{\bar{u}}{u}, \ \mu = \frac{\bar{\mu}}{\mu_0}e^{-\frac{E}{RT}}, \ \theta = \frac{E(\bar{T} - T_0)}{RT_0^2}, \ \in = \frac{RT_0}{E}, \ \Omega = \frac{E}{RT_0},$$

$$Br = \frac{\mu_0 EU}{kRT_0^2}, \ \gamma = \frac{\mu_0 EU}{kRT_0^2}e^{-\frac{E}{RT_0}}, \ H^2 = \frac{\sigma B_0^2 L^2}{\rho} \ \text{and} \ \lambda = \frac{QC_0EAL^2}{KRT_0^2}e^{-\frac{E}{RT_0}} \qquad (5)$$

where (*x, y*) are distances measured in stream wise and normal directions, respectively, μ is the dynamic viscosity, *k* represents the thermal conductivity, *u* is the axial velocity, *U* is the velocity scale, $\in$ is the activation energy parameter, λ is the Frank-Kamenettski parameter. However, the flow at the upper plate is driven by a constant velocity without the pressure gradient.

$$\frac{d}{dy}(\mu\frac{du}{dy}) - H^2u = 0 \qquad (6)$$

$$\frac{d^2\theta}{dy^2} + \lambda\left(e^{\frac{\theta}{1+\in\theta}} + \gamma\left(\mu\left(\frac{du}{dy}\right)^2 + H^2u^2\right)\right) = 0 \qquad (7)$$

with the appropriate boundary conditions as follows:

$$u(0) = \theta(0) = 0 \quad \text{and} \quad u(1) = 1 \quad \text{and} \quad \theta(1) = 0 \qquad (8)$$

such that, the dynamic viscosity is given as $\mu = e^{-\frac{\theta}{1+\in\theta}}$.

Solution Method

The governing Equations (6) and (7) are non–linear differential equations (ODE) with no exact analytical solution. Therefore, for accuracy, effectiveness, better stability and simplicity, we employed the use of series solution of modified Adomian decomposition method by assuming that the outcomes appear in the following forms:

$$u(y) = \sum_{n=0}^{\infty} u_n(y) \ \text{and} \ \theta(y) = \sum_{n=0}^{\infty}\theta_n(y) \qquad (9)$$

$$u(y) = a_0y + \int_0^y\int_0^y\left\{H^2ue^{\frac{\theta}{1+\in\theta}} - \in\frac{\theta}{(1+\in\theta)^2}\frac{d\theta}{dy}\frac{du}{dy}\left(\frac{1}{1+\in\theta}\right)\frac{d\theta}{dy}\frac{du}{dy}\right\}dYdY \qquad (10)$$

$$\theta(y) = b_0 y - \lambda \int_0^y \int_0^y \left\{ e^{\frac{\theta}{1+\in\theta}} + \gamma \left[e^{-\frac{\theta}{1+\in\theta}} \left(\frac{du}{dy} \right)^2 + H^2 u^2 \right] \right\} dY dY \qquad (11)$$

where a_0 and b_0 are constants of integration to be resolved by putting to use the limit conditions in Equation (8). With the series solutions introduced in Equation (9), the Equations (10) and (11) reduce to the following:

$$u(y) = a_0 y + \int_0^y \int_0^y \left\{ H^2 \sum_{n=0}^{\infty} A_n - \in \sum_{n=0}^{\infty} B_n - \sum_{n=0}^{\infty} C_n \right\} dY dY \qquad (12)$$

$$\theta(y) = b_0 y - \lambda \int_0^y \int_0^y \left\{ \sum_{n=0}^{\infty} D_n + \gamma \left[\sum_{n=0}^{\infty} E_n + H^2 \sum_{n=0}^{\infty} F_n \right] \right\} dY dY \qquad (13)$$

where the non-linear terms present are termed respectively as follows:

$$\sum_{n=0}^{\infty} A_n(y) = \sum_{n=0}^{\infty} u_n e^{\frac{\sum_{n=0}^{\infty}\theta_n}{1+\in \sum_{n=0}^{\infty}\theta_n}}, \quad \sum_{n=0}^{\infty} B_n(y) = \frac{\sum_{n=0}^{\infty}\theta_n}{\left(1+\in \sum_{n=0}^{\infty}\theta_n\right)^2} \frac{d\sum_{n=0}^{\infty}\theta_n}{dy} \frac{d\sum_{n=0}^{\infty}u_n}{dy},$$

$$\sum_{n=0}^{\infty} C_n(y) = \left(\frac{1}{1+\in \sum_{n=0}^{\infty}\theta_n} \right) \frac{d\sum_{n=0}^{\infty}\theta_n}{dy} \frac{d\sum_{n=0}^{\infty}u_n}{dy}, \quad \sum_{n=0}^{\infty} D_n(y) = e^{\frac{\sum_{n=0}^{\infty}\theta_n}{1+\in \sum_{n=0}^{\infty}\theta_n}},$$

$$\sum_{n=0}^{\infty} E_n(y) = e^{-\frac{\sum_{n=0}^{\infty}\theta_n}{1+\in \sum_{n=0}^{\infty}\theta_n}} \left(\frac{d\sum_{n=0}^{\infty}u_n}{dy} \right) \quad \text{and} \quad \sum_{n=0}^{\infty} F_n(y) = \sum_{n=0}^{\infty} u_n^2 . \qquad (14)$$

such that the Adomian polynomials, A_0, A_1, A_2,……. to F_0, F_1, F_2, …… are obtained by expanding each expression in Equation (14) such that few are stated thus:

$$A_0 = u_0(y) e^{\frac{\theta_0(y)}{\in\theta_0(y)+1}}, \quad A_1 = \frac{e^{\frac{\theta_0(y)}{\in\theta_0(y)+1}} \left(u_0(y)\theta_1(y) + u_1(y)(\in\theta_0(y)+1) \right)^2}{\left(\in\theta_0(y)+1\right)^2}, \ldots,$$

$$B_0 = \frac{\theta(y) u_0'(y) \theta_0'(y)}{\left(\in\theta_0(y)+1\right)^2},$$

$$B_1 = \frac{\left(u_0'(y)\theta_1(y)\theta_0'(y)(1-\in\theta_0(y)) + \theta_0(y)(\in\theta_0(y)+1)\right)\left(u_1'(y)\theta'_0(y) + u_0'(y)\theta_1'(y)\right)}{\left(\in\theta_0(y)+1\right)^3}, \ldots$$

$$C_0 = \frac{u_0'(y)\theta_0'(y)}{\in\theta_0(y)+1}, \quad C_1 = \frac{(\in\theta_0(y)+1)\left(u_1'(y)\theta'_0(y) + u_0'(y)\theta_1'(y) - \in\theta_0'(y)u_0'(y)\theta_1(y)\right)}{\left(\in\theta_0(y)+1\right)^2}, \ldots\ldots..$$

$$D_0 = \frac{\theta_0(y)}{e^{\in\theta_0(y)}+1}, \quad D_1 = \frac{\theta_1(y) e^{\frac{\theta_0(y)}{\in\theta_0(y)+1}}}{\left(\in\theta_0(y)+1\right)^2}, \ldots, \quad E_0 = u_0'^2(y) e^{-\frac{\theta_0(y)}{\in\theta_0(y)+1}},$$

$$E_1 = \frac{u_0'(y) e^{-\frac{\theta_0(y)}{\in\theta_0(y)+1}} \left(2u_1'(y)(\in \theta_0(y)+1)^2 - \theta_1(y)u_0(y)\right)}{\left(\in \theta_0(y)+1\right)^2}, \ldots, \; F_0 = u_0(y),$$

$$F_1 = 2u_0(y)u_1(y), \ldots \quad (15)$$

With the Adomian polynomials obtained in Equation (15), we take the zeroth components of the couple Equations (12) and (13) as described in Wazwaz and El-Sayed (2001), and Hassan et al. (2017) for the solutions as follows:

$$u_0 = a_0(y), \quad \theta_0 = 0 \quad (16)$$

$$u_1(y) = \int_0^y\int_0^y \left\{H^2 A_0(y) - \in B_0(y) - C_0\right\} dYdY \quad (17)$$

$$\theta_1(y) = b_0 y - \lambda \int_0^y\int_0^y \left\{D_0(y) + \gamma\left[E_0(y) + H^2 F_0\right]\right\} dYdY \quad (18)$$

$$u_{n+1}(y) = \int_0^y\int_0^y \left\{H^2 A_n(y) - \in B_n(y) - C_n(y)\right\} dYdY \qquad \forall n \geq 1 \quad (19)$$

$$\theta_{n+1}(y) = -\lambda \int_0^y\int_0^y \left\{D_n(y) + \gamma\left[E_n(y) + H^2 F_n(y)\right]\right\} dYdY \qquad \forall n \geq 1 \quad (20)$$

Equations (16) to (20) are hereby done repeatedly to acquire the approximate results for momentum and energy distributions as:

$$u(y) = \sum_{n=0}^{k} u_n(y) \text{ and } \theta(y) = \sum_{n=0}^{k} \theta_n(y) \quad (21)$$

Entropy Generation

Flow and heat transfer processes within parallel channels interact with each other and thereby causing the exchange of thermal energy within the flow stream, thus resulting in the rate of entropy production. Following Wood (1975), Bejan (1996), Makinde (2008), and Hassan and Gbadeyan (2015), the equation for finding the rate of entropy generation together with the influence of magnetic strength is presented as:

$$S^m = \frac{k}{T_0^2}\left(\frac{d\bar{T}}{d\bar{y}}\right)^2 + \frac{\bar{\mu}}{T_0}\left[\left(\frac{\partial \bar{u}}{\partial \bar{y}}\right)^2 + \sigma\frac{\beta_0}{\rho}\bar{u}^2\right] \quad (22)$$

The terms in Equation (22) show the irreversibility caused by heat transfer and the local entropy production caused by viscous dissipation and magnetic strength, respectively. Using the existing parameters and variables in Equation (5), the rate of entropy generation is given as:

$$N_s = \frac{S^m L^2 T_0^2}{T_0^2} = \left(\frac{d\theta}{dy}\right)^2 + \frac{Br}{\Omega}\left[\mu\left(\frac{\partial u}{\partial y}\right)^2 + H^2u^2\right] \quad (23)$$

For convenience, Equation (23) is divided into two portions, N_1 and N_2 to respectively represent the irreversibility due to heat transfer and effects of viscous dissipation together with magnetic strength. With that we have

$$N_1 = \left(\frac{d\theta}{dy}\right)^2 \quad \text{and}$$

$$N_2 = \frac{Br}{\Omega}\left[\mu\left(\frac{\partial u}{\partial y}\right)^2 + H^2u^2\right] \quad (24)$$

However, we defined the irreversibility distribution ratio as:

$$\phi = \frac{N_1}{N_2} \quad (25)$$

such that heat transfer dominates when $0 < \phi < 1$ and fluid friction dominates when

$\phi > 1$. As an alternative to irreversibility parameter, the Bejan number (*Be*) is defined as

$$Be = \frac{N_1}{N_s} = \frac{1}{1+\phi} \text{ where } 0 \le Be \le 1 \quad (26)$$

Results and Discussion

In this section, we describe the outcomes from the coupled differential equations to determine the impact of magnetic strength on a temperature-dependent reactive variable viscosity Couette flow. Table 1 displays the rapid convergence of the series solution for the constants a_0 and b_0 in Equations (10) and (11) with little iteration to show the effectiveness of the series solutions of modified Adomian decomposition technique as already discussed in introduction. Table 2 displays the comparison of the numerical outcomes of temperature distribution obtained using perturbation method in investigation done in Kobo and Makinde (2010) and the present results obtained from modified decomposition method of Adomian (mADM). The results obtained from both methods demonstrate the agreement between the two methods with difference of order 10^{-3}.

Table 1: Rapid convergence of the series solution for a_0 and b_0

$\gamma = \lambda = H = \in = 0.1$		
n	a_0	b_0
0	1	0
1	0.99834	0:054992
2	1.00777	0:055551
3	1.00760	0:055426
4	1.00758	0:055428
5	1.00758	0:055428

Table 2: Comparison of numerical outcomes of the temperature profile

$\boldsymbol{H = 0, \epsilon = 0.1, \gamma = 0.1, \lambda = 0.5}$			
Y	T(y)PM (Kobo and Makinde 2010)	θ $(y)mADM$	Absolute error
0	0		0
0.1	0.02360044943	0.02475	1.14955×10^{-3}
0.2	0.04208380022	0.04400	1.91620×10^{-3}
0.3	0.05535532038	0.05775	2.39468×10^{-3}
0.4	0.06334613033	0.06600	2.65387×10^{-3}
0.5	0.06601440811	0.06875	2.73559×10^{-3}
0.6	0.06334613023	0.06600	2.65387×10^{-3}
0.7	0.05535532037	0.05775	2.39468×10^{-3}
0.8	0.04208380020	0.04400	1.91620×10^{-3}
0.9	0.02360044944	0.02475	1.14955×10^{-3}
10	0	0	0

Figure 2 displays the fluid motion with variation in magnetic strength parameter (H). It is noticed that the flow speed remarkably reduces as (H) rises. This is caused due to the presence of electromagnetic force across the channel and thus brings about the delay in the fluid flow.

The temperature profiles of the fluid system are with variations in the values of magnetic strength parameter (H), Frank-Kamenettski parameter (λ) and viscous heating parameter (γ) are respectively displayed in Figures 3 to 5. The greatest temperature occurs at the centerline of the flow regime on a general note. It is noticed that the maximum temperature occurs with the least value of (H) in Figure 3 and otherwise gives the greatest value of (λ) in Figure 4 and in Figure 5. The increment in fluid temperature in Figure 4 and 5 is due to the overwhelming performance of variable viscosity properties that is an exponential relation of temperature.

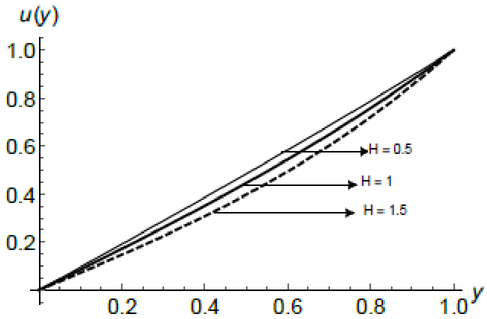

Figure 2: Effect of H on $u(y)$

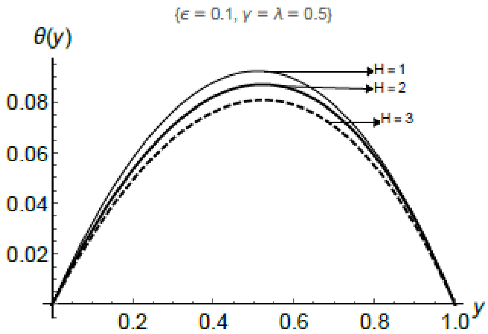

Figure 3: Effect of H on $\theta(y)$

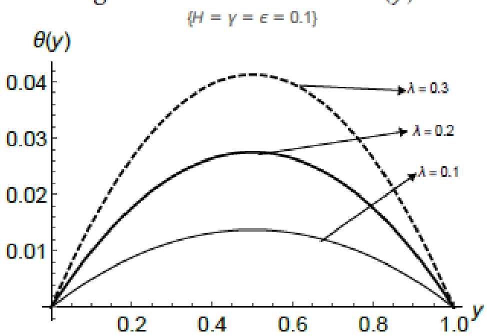

Figure 4: Effect of λ on $\theta(y)$

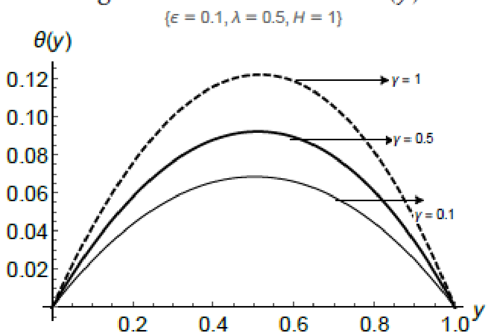

Figure 5: Effect of γ on $\theta(y)$

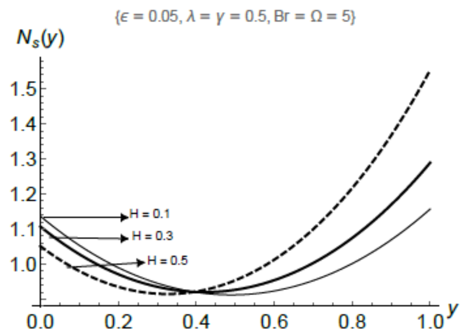

Figure 6: Effect of H on $N_s(y)$

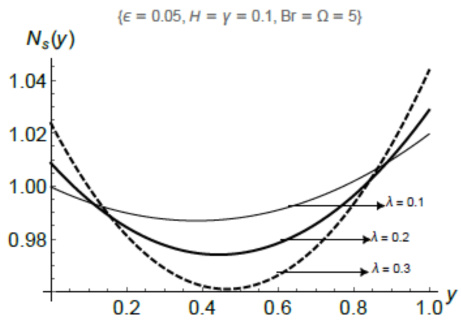

Figure 7: Effect of λ on $N_s(y)$

The rate of entropy generation distribution for different values of magnetic strength parameter (H), Frank-Kamenettski parameter (λ) and viscous heating parameter (γ) are respectively illustrated in Figures 6 to 8. The rate of entropy distribution is generally active at both upper and lower walls throughout the flow stream within the channel. The rate of disturbance reduces at lower plate and increases at the upper plate but maintain equilibrium at the centreline with rising values of H in Figure 6. However, the rate of entropy distribution increases at both the lower and upper plates but otherwise at the centreline with the increasing values of (λ) in Figure 7 and (γ) in Figure 8.

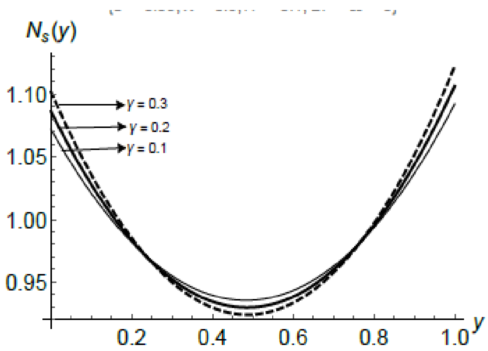

Figure 8: Effect of γ on $N_s(y)$

Conclusion

The study conducted herein showed the significant effect of magnetic strength on a temperature dependent variable viscosity of reactive hydromagnetic coquette flow with fixed lower wall and moving upper wall that are both parallel. The coupled differential equations controlling the fluid flow are determined using the modified decomposition method of Adomian (mADM). The results obtained for momentum and energy distributions are used to determine the rate of entropy generation, the irreversibility distribution ratio and Bejan number. The results showed the meaningful impact of magnetic strength that reduces the fluid momentum together with fluid temperature. Also, the exponentially temperature-dependent variable viscosity contributed to the remarkable increase in fluid temperature with respect to viscous heating parameter and combustion parameter.

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