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Mathematical Programming Model for the Two-Level Facility Location Problem: The Case of Tanzanian Emergence Maize Distribution Network for 2004–2010 Maize Data

Said A. Sima
Department of Mathematics, University of Dar es Salaam, Box 35062, Dar es Salaam, Tanzania
E-mail: said.sima@gmail.com
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Abstract

A two-level facility location problem (FLP) has been studied in the transportation network of emergence maize crop in Tanzania. The facility location problem is defined as the optimal location of facilities or resources so as to minimize costs in terms of money, time, distance and risks with the relation to supply and demand points. Distribution network design problems consist of determining the best way to transfer goods from the supply to the demand points by choosing the structure of the network such that the overall cost is minimized. The three layers, namely production centres (PCs), distribution centres (DCs) and customer points (CPs) are considered in the two-level FLP. The flow of maize from PCs to CPs through DCs is designed at a minimum cost under deterministic mathematical programming model. The four decisions to be made simultaneously are: to determine the locations of DCs (including number of DCs), allocation of CPs to the selected DCs, allocation of selected DCs to PCs, and to determine the amount of maize crop transported from PCs to DCs and then from DCs to CPs. The modelled problem generated results through optimization with respect to optimal location-allocation strategies. The results of the optimized network shows the improvement in costs saving compared to the manually operated existing network. The results show the costs saving of up to 18% which is equivalent to $2,910 thousand (TZS 2.9 billion).

Keywords: Optimization, Maize crop, Transportation network, Deterministic model, Facility location.

Introduction

Distribution network design problems consist of determining the best way to transfer goods from the supply to the demand points by choosing the structure of the network such that the overall costs are minimized (Ambrosino and Scutella 2005). Here, the network is considered from a graph theory point of view where a connected graph has sets of vertices (nodes) and edges (arcs). In this context, production centres, warehouses (distribution centres) and customer points/demand points are assumed to be vertices, while edges or arcs can act as roads and/or railways or other pathways such as marine routes or flight routes.

In the facility location problem (FLP), it is required to determine the optimal location of facilities or resources so as to minimize costs in terms of money, time, distance and risks in relation to supply and demand points (Ahuja et al 1993, Nagy and Salhi 2007). Sajjadi defined FLP as follows: - "given a set of facility locations and a set of customers who are served from the facilities, then which facilities should be used? Which customers should be served from which facility so as to minimize total cost of serving all customers?” (Sajjadi 2008). Some examples of such facilities are schools, warehouses, hospitals, markets, industries, stadium or open space, terminal bus stand (hub), railway stations, military centres, post offices, fire stations and worship places (Cox 1998, Klose and Drexl 2005, Sajjadi 2008).

The FLP is a broad study area within the location analysis, where the location, allocation and shipment or transportation decisions are solved simultaneously. Usually, the allocation of customers to a specific facility has implication to transportation of goods from that facility to the respective customers. In this context, each customer is supplied directly from a facility without depending on other customer's demands (Klose and Drexl 2005, Ameli et al 2009, Melo et al 2009). The FLP is said to be a two-level problem when there are two transportation levels; from PCs to DCs and from DCs to CPs.

This work studied a two-level FLP so as to optimize the costs of the network which is important for food security at the customers' demand locations. The motivation for this study is fact that the existing literatures in FLP do not address the maize crop distribution network in Tanzania (Snyder 2006, Nagy and Salhi 2007) to the best of knowledge. The objective is to come up with food distribution system that is economical and cost effective for emergence maize distribution in Tanzania. The network is useful during an emergency situation to rescue areas with food shortage. The main task is to analyse the existing distribution set-up in order to study whether it is optimal in terms of cost. This network has five existing DCs where it is possible to vary their capacities during optimization. The flow of maize from PCs to CPs through DCs is optimized. There are two tasks in the analysis of the existing network: Firstly, the optimization of the flow of maize in the existing distribution network. Through optimization of the model, the optimal solution obtained will be compared to the cost of manually operated network. In this case, capacities of DCs will be considered as constant terms. Secondly, the improvement of the existing network through optimization tools is studied. The aim is to satisfy the customers' demand while minimizing the overall network cost. The same five DCs are used but with variable capacities in order to find the best capacities with minimum cost. The capacities in this case are considered as decision variables.

The results from optimization of the existing network will help to recommend cost reduction measures to the Tanzanian government with regard to emergency maize distribution. The main objective is to determine whether to keep the current maize distribution network or to use a different network. Scientific methods, particularly optimization results are used to give an answer to the raised question. The problem is modelled and solved with respect to optimal location-allocation strategies. Specific objectives are as follows:

- To develop a deterministic mathematical programming model for two-level FLP;
- To apply the developed model to Tanzanian emergence maize distribution network;
- To analyse and compare the results obtained.

This study is useful as it will provide a mechanism for reducing food prices within the country during disasters.

A two-level facility location problem for maize distribution network in Tanzania

The Ministry of Agriculture, Food Security and Cooperatives (MAFSC) of Tanzania, is responsible for food security. The major cereal crops produced in the country are maize (corn), rice (paddy), millet, finger-millet, sorghum and wheat. The country's major food crops (main staple crops) are maize and rice. The problem considered in this paper is maize production and its distribution system in Tanzania. This research considered only maize distribution as per available data, and also maize is the only food crop which is stored in the DCs and is managed by the National Food Reserve Agency (NFRA) under the MAFSC for emergency situations (URT MAFSC 2013).

The emergency situations considered are acute food shortage in some places in the country (due to drought and other disasters). The stored crop helps in price stabilization in the markets, especially in the urban areas. In the country, there are some common deficit zones due to drought and other weather effects like shortage of rainfall in semi-arid areas.

The food crops production in the country is highly concentrated in the southern highlands regions (Rukwa, Katavi, Songwe, Njombe, Mbeya, Iringa, Morogoro and Ruvuma) and the peripheral areas of the country as shown in Figure 1. On the other hand, the traditional food deficit areas are located mostly in the central corridor regions (Singida, Dodoma and Tabora) and northern parts (Arusha, Manyara, Kilimanjaro and Tanga), and other parts as shown in the map of Tanzania (Figure 1). The specific locations of existing DCs (warehouses) are also shown in Figure 1. The DC in this context is a storage building where commodities are stored for some time before being taken to customers. The specific demand points (customer points); are not shown in the map for clarity; rather some major demand zones have been marked, but the production centres are within the marked production zones, particularly in the southern highlands.

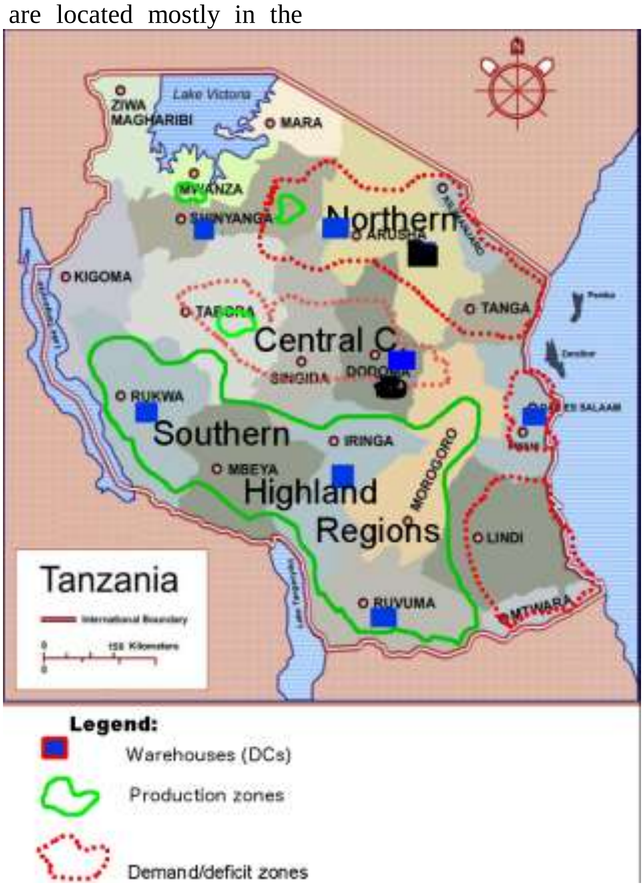

Figure 1: The map of Tanzania showing the food production zones, DCs and demand zones.

Major maize surplus production is from the four regions (known as `The Big four') namely, Rukwa, Mbeya, Iringa and Ruvuma (Mkenda and Campenhout, 2011). However, these regions had been subdivided into other new regions that include Njombe, Songwe and

Katavi. This study considers original four regions to concur with the collected data. The specific PCs form a production zone in the southern highlands part of the country. In this study, the PCs will form the first layer among the three layers in the two-level FLP. The maize crops are bought from this production zone by the NFRA for storage in the DCs.

The DCs are scattered in different parts of the country. Usually the storage in the DCs is done for a year (a harvest season to the next harvest season). There are seven existing DCs with a total capacity of 241 thousand tons. These are Arusha (39 thousand tons capacity), Dar es Salaam (52 thousand tons), Dodoma (39 thousand tons), Shinyanga (14.5 thousand tons), Makambako-Iringa (34 thousand tons), Songea (24 thousand tons) and Sumbawanga (38.5 thousand tons). These DCs, as shown in Figure 1, form the second layer of the two-level FLP model of the study. The first five DCs are used for storage of maize to be supplied to the deficit CPs throughout the country. The last two DCs, Sumbawanga and Songea, are used as reserve DCs to buffer the other five DCs. These two DCs are located in the production zones. The third layer of the model in this study is CPs. These are specific demand points in the country to be supplied by DCs during food deficit time. As indicated in Figure 1, the major deficit zones are central corridor zone and the northern zone. The three layers form the distribution system that needs to be designed at minimum cost while satisfying the customers' demands.

In Tanzania, physical access to food is affected by inadequate transportation infrastructure. Due to long distances between the food PCs, DCs and CPs, together with inadequate and unreliable transportation network to some places, high transportation costs are among the challenges. For instance, the existing distribution system has distances ranging from120 kilometres (km) to 1,348 km between PCs and DCs. The distances between DCs and CPs, on average, also range from 136 km to 360 km. This results, at times, to high food prices in deficit areas, and therefore affects access to food by both low incomes, rural and urban populations (Mkenda and Campenhout 2011, USAID 2011). Table 1 gives the summary of PCs, DCs and CPs; and their location zones within the country. The southern highlands zone is the only zone with PCs and also 3 DCs out of the 7 DCs. Notably, based on 2010 data, the Ruvuma, Rukwa, Kigoma and the Dar es Salaam regions have no CPs. All the mentioned regions except Dar es Salaam are self-sufficient in cereal crops production and the surplus production is always expected. The Dar es Salaam region is the place of city dwellers that is populated mostly by employed people who earn salaries. Food deficiencies are mostly realized by people living in the rural areas. The list of DCs and their respective CPs of the collected actual distribution data from 2004 to 2010 were obtained from NFRA.

Table 1: PCs, DCs and CPs distribution in the country (Source: NFRA, Prime Minister Office)

Zone	Specific Regions	# of PCs	# of DCs	# of CPs
Southern Highlands	Iringa, Rukwa, Mbeya, Ruvuma	4	3	9
Central Corridor	Dodoma, Singida,Tabora	0	1	17
Northen	Arusha, Manyara, Tanga, Kilimanjaro	0	1	24
Southern Corridor	Mtwara, Lindi	0	0	8
Eastern	Dar es Salaam, Coast, Morogoro	0	1	11
Lake Victoria	Shinyanga, Mwanza Mara, Kagera, Kigoma	0	1	24
	Total	**4**	**7**	**93**

Materials and Methods

Mathematical programming model for the two-level facility location problem

This section presents the deterministic mathematical model of two-level FLP. Here, a deterministic model with a single-product and a single-period planning horizon is considered. The aim is to design a deterministic capacitated two-level FLP and to optimize location, allocation and hence transportation decisions for the distribution network. The deterministic mathematical model for a single period demand consists of four months which are from January to April in each year. The model is adapted from Elhedhli and Goffin (2005) and other references (Azad and Davoudpour 2013, Hindi et al. 1998, Klose and Drexl 2005, Lashine et al 2006, Sahin and Sural 2007).

The notations used in the model are as follows:

J: is the index set for PCs, where j ϵ J and |J| denotes the total number of PCs, i.e., PC_j, $j \in J$. PC_j = PC allocated at site j.

K: is the index set for DCs, where k ϵ K and |K| denotes the total number of possible DC sites, i.e., DC_k = DC located at site k, k ϵ K.

L: is the index set for CPs, where l ϵ L and |L| denotes the total number of CPs, i.e., CP_l, l ϵ L. CP_l = CP located at site l, l ϵ L. CP_l has fixed location together with their associated demand, D_l.

R_k: the set of capacities of given DC_k. Hence $R_k = \left\{V_k^1,\ V_k^2,\ldots,V_k^{|R_k|}\right\}$ where $V_k^{|R_k|}$ is possible set of capacities for given DC_k. V_k^r is possible set of capacities of given DC_k.

S_j : Supply (production capacity) of a maize crop at PC_j .

D_l: Total demand for four months for maize crop at CP_l transported only once in a week. This amount was considered to be transported in the first week of the four months period (January to April) of a year.

F_k^r: Total fixed annual operating cost in US dollar for a DC with $V_{k,}^r$, i.e., r ϵ{1, 2, …, $|R_k|$}, where V_k^r is possible set of capacities of given DC_k.

C_{jk}: A road distance in kilometres from PC_j to DC_k, j ϵ J, k ϵ K.

T_{kl}: A road distance in kilometres from DC_k to CP_l, k ϵ K, l ϵ L.

λ: This is a unit cost for transferring 1 ton of maize crop for a 1 km distance, and the cost is in $ (per km per ton).

Decision variables for the model:

X_{jk}: Amount in tons flow from PC_j to DC_k.

Y_{kl}: Amount in tons flow from DC_k to CP_l.

Z_k^r: A binary location variable that will be 1 if a DC_k is selected with a capacity $V_{k,}^r$ and 0 otherwise. When a single capacity per DC_k is used we ignore the superscript r in Z_k^r, V_k^r and $F_{k,}^r$. Here the choice of capacity is not a decision variable but the choice of site k is.

The resulting mixed integer linear programming can then be formulated as:

$$\min_{X_{jk}, Y_{kl}, Z_k} \lambda\left[\sum_j \sum_k C_{jk} X_{jk} + \sum_k \sum_l T_{kl} Y_{kl}\right] + \sum_k F_k Z_k \quad (1)$$

subject to

$$\sum_k X_{jk} \le Sj,\ \forall\, j \in J \quad (2)$$

$$\sum_j X_{jk} = V_k Z_{k,}\ \forall k \in K \quad (3)$$

$$\sum_l Y_{kl} \le V_k Z_{k,} \forall k \in K \quad (4)$$

$$\sum_k Y_{kl} = D_{l,} \forall l \in L \quad (5)$$

$$X_{jk} \ge 0, \forall j,k;\ j \in J,\ k \in K \quad (6)$$

$$Y_{kl} \ge 0, \forall k,\ l,\ k \in K, l \in L \quad (7)$$

$$Z_k \in \{0,1\}, \forall k \in K. \quad (8)$$

Where:

- The objective function (1) minimizes the total distribution cost, e.g. transportation cost from PCs to DCs and DCs to CPs, and fixed annual operation costs, F_k, for DCs and the corresponding capacities V_k.

- Constraints (2) are the supply constraints (PCs' capacities), where the amount to be transported from a PC_j to the selected DCs, must not exceed its capacity, S_j.
- Constraints (3) refer to the amount supplied from PC_j to all selected DC_k, must not exceed the DCs' capacity V_k.
- Constraints (4) refer to the amount supplied by DC_k, to all CP_l, $l \in L$, without exceeding V_k. V_k (respectively F_k) are the values currently used in the current transportation network with five DCs, e.g. DC_1, …., DC_5. The capacities V_k, $k \in K$, are not necessarily equal.
- Constraints (5) represent the amount to be transported from all DC_k, $k \in K$, to the CP_l, must meet a demand, D_l, at the CP_l.
- Constraints (6) and (7) are the non-negativity restrictions.
- Constraints (8) are binary variables.

In the optimization results, four decisions are sought as follows:-

(a) Location decisions: Where and how many DCs to locate out of |K|? The optimal decisions to be made here are the number of DCs and their physical locations (i.e., values of Z_k or Z_k^r in the case of multiple capacities).

(b) Allocation decisions: Which DCs to be served by which PCs (i.e., the pair (PC_j ;DC_k); $j \in J$; $k \in K$) and which CPs are to be served by which selected DCs (i.e., the pair (DC_k: CP_l); $k \in K$; $l \in L$)? The optimal results will give the allocations of DCs to PCs and CPs to DCs simultaneously.

(c) Transportation decisions: From location and allocation decisions, what is the amount to be transported from PCs to DCs (i.e. values of X_{jk}) and DCs to CPs (i.e. values of Y_{kl})? The transported amounts, X_{jk} and Y_{kl} will be determined. Hence, direct shipment routes designing from PCs to DCs and also from DCs to CPs will be established.

(d) Capacity value decisions: What is the best capacity for each DC_k to be selected from several possible capacities, $\{V_k^1, \ldots, V_k^{|R_k|}\}$. For the case of a single capacity per DC_k the decision variables Z_k will suffice.

Data, Results and Discussion

Data used in this research is taken from Tanzania official records (Sima 2015), where the three layers used in the model are PCs and CPs. Road connections to the three layers form the production, storage and distribution network. The total capacity of all four PCs, $\sum_{j=1}^{4} S_j$, is 532,000 tons. These data are based on annual production capacity of 2011/2012. The total capacity of the five DCs, $\sum_{k=1}^{5} V_k$ is 178,500 tons.

The CPs are classified as 93 districts as obtained from 2004 to 2010 maize distribution data. This data was collected from the head of the disaster management unit in the Prime Minister's office, in January 2011. There are 93 CPs used in this study with total demand, $\sum_{l=1}^{93} D_l$, of 145,144 tons. There are five DCs each having fixed capacities, $\widehat{V_k}$. The DCs and PCs will be denoted in terms of their indices as shown in Table 2. The computational experiments consider the cases; 1 and 2 as explained in next paragraphs.

Table 2: Notations for DCs and PCs

DCs	DC_k	PCs	PC_j
Dar es Salaam	DC_1	Iringa	PC_1
Arusha	DC_2	Mbeya	PC_2
Dodoma	DC_3	Rukwa	PC_3
Makambako	DC_4	Ruvuma	PC_4
Shinyanga	DC_5		

There are several common inputs to be used in Cases 1 and 2. These are $|J| = 4$, $|K| = 5$, and $|L| = 93$. Other common inputs are the PCs' fixed capacities, S_j, distances C_{jk} and T_{kl}. The CPs' demands, D_l, l = 1; 2,...,$|L|$, are given as inputs to the model in all computational experiments. Also the unit transportation cost λ = $0.10795 (per km per ton) was used based on 2010 conversion rate between Tanzanian currency and US $. The unit transportation cost estimated is based on NFRA (National Food Reserve Agency) maize crop transportation cost in 2010. These were available data as specified to this paper.

Generally, the research data are from four sources, which are; the Tanzania National Roads Agency (TANROADS), Ministry of Agriculture, Food Security and Cooperatives (MAFSC), National Food Reserve Agency (NFRA) and the disaster management department in the Prime minister's office.

The CPLEX software (IBM ILOG Optimization studio) was used for all the computational experiments. There is no any stopping criteria imposed; and the program will stop when it cannot enumerate any more improved solutions.

Case 1: DCs' single capacity and computational results

Here, the five DC sites were considered together with their current capacities, R_k = $\{\widehat{V_k}\}$. In this case, the optimization was performed with respect to the decisions (a), (b) and (c) as listed in the previous section. The model stated by (1)–(8) was used for optimization. The purpose of this case was to see if the current network is optimal.

The optimized results are summarized in Table 3 where the first 5 columns contain some inputs to the model. The last 5 columns in Table 3 present the results obtained from optimization. For example, the variable Z_k is used to show if the corresponding DC has been selected. The selected DCs have to be supplied from the PCs and this has been shown in column 7. The notation PC_j^s in this column is used to denote the PC_j that are the suppliers of the respective DCs. For example, it can be seen from the first entry of column 7 that PC_1 and PC_2 served the DC_1. Column 8 shows the amount of supplies received by each DC from the corresponding PCs. For example, the second entry of column 8 shows that DC_2 received 39,000 tons from PC_1.

A comparison of columns under $\widehat{V_k}$ and $\sum_j X_{jk}$, is that the full capacity of each DC is utilized.

Table 3: Location allocation results for true capacity in Case 1

DC_k	$\widehat{V_k}$	F_k	PC_j	S_j	Z_k	PC_j^s	$\sum_j X_{jk}$	$\lvert L_k \rvert$	$\sum_{l=1}^{Lk} Y_{kl}$
DC_1	52,000	340,340	PC_1	100,000	1	PC_2 ; PC_1	22,000 ;30,000	28	39,361
DC_2	39,000	255,260	PC_2	251,000	1	PC_1	39,000	18	39,000
DC_3	39,000	255,260	PC_3	140,000	1	PC_1	39,000	31	39,000
DC_4	34,000	222,530	PC_4	41,000	1	PC_2	34,000	11	13,283
DC_5	14,500	94,900			1	PC_2	14,500	10	14,500
Total	178,500	1,168,290		532,000			178,500	98	145,144

The notation L_k has been introduced to denote the set of CPs (where $|L_k|$ is the number of CPs) served by DC_k. Hence, the total shipment to these CPs from the DC_k is $\sum_{l=1}^{|L_k|} Y_{kl}$, shown in the very last column. For example, DC_1 supplied a total of 39,361 tons to 28 CPs which it is within its capacity. Note that, a CP can get supply from more than one DC under the so called multi-sourcing. Hence, $\sum_{k=1}^{|K|} |L_k| \geq |L|$, as can be seen at the last entry in column under $|L_k|(\sum_{k=1}^{|K|}|L_k| = 98 > |L|)$. In the results, the total demands of 145,144 tons from all the CPs are satisfied (see the total value at the last row and last column, Table 3).

With respect to the location decision, Table 3 shows that all the five DCs have been selected as shown in the column under Z_k. Also with respect to the allocation decision, it can be seen in Table 3 that all the five DCs are supplied by only PC_1 and PC_2 as shown in the column under PC_j^s. This clearly shows that the existing network results are different from the manually operated system since the two PCs (PC_3 and PC_4) are never used. This is based on the fact that the current network has been using PC_3 and PC_4 as shown in Table 4. Data in Table 4 were prepared using data obtained from NFRA. PC_1 and PC_2 are the largest producers among the four PCs as shown in Table 5. This table shows the different annual production capacities for all the four PCs. Table 3 also shows that PC_1 is the only PC to supply its full capacity to the DCs.

In addition, results obtained are not the same as the data used in the current network with regard to shipments between DCs to CPs. This can be seen by comparing the results under $|L_k|$ in Table 3 with the data in the last column of Table 4.

Table 4: PCs to DCs supplies from manually operated current network

DC_k	V_k	PC_j	PCs	X_{jk}	$\lvert L_k \rvert$
DC_1	52,000	PC_1	PC_1; PC_3 ; PC_4 PC4	6,305; 61; 153	26
DC_2	39,000	PC_2	PC_3	9, 867	16
DC_3	39,000	PC_3	PC_3	4,009	12
DC_4	34,000	PC_4	PC_3	7,523	9
DC_5	14,500				30
Total	**178,500**			**27,918**	**93**

Table 5: The summary of PCs annual maize crop total production capacity in tons

PC_j	Year					Average
	2005/06	**2006/07**	**2007/08**	**2008/09**	**2009/10**	
PC_1	412,762	474,270	384, 273	443, 905	393,164	421,675
PC_2	293,725	349,094	494,810	393,406	621,545	430,516
PC_3	270,564	226,524	351,013	375,732	372,830	319,333
PC_4	211,789	138,269	236,602	176,876	289,588	210,625

In Case 1, the total distribution cost that includes transportation costs and DCs' annual fixed operation cost is $15,570,885.08. This is the minimum objective value obtained from optimization after 27 seconds. In Case 1, all DCs use $\overline{V}_k$ as their true capacities. However, at times the demand at CPs increases and therefore replenishment is needed at DCs in order to cater for the additional demand at CPs. The capacity of DC_5 is an example of this situation as it can be seen by comparing its capacities shown in Tables 3 and 6. The replenishment is carried out from the two PCs, Songea and Sumbawanga. On the other hand, the actual capacity used may not exceed the true capacity. Therefore, the true capacity and the capacity used (actual capacity) may not be the same. Since the capacity used by DC_k varies from year to year, the maximum actual capacity, $\widehat{V_k}$ used by DC_k during 2004–2010 was taken. This actual capacity for the existing network is also considered as the manually operated existing distribution network. The program had to be re-run using $\widehat{V_k}$ instead of $\overline{V_k}$ (true capacity) and results are summarized in Table 6. Other inputs to the model (1)–(8) remained the same. Columns of Table 6 contain the same headings as in Table 3. The results were analysed with respect to the three decisions which are location, allocation and transportation.

In the location decision, all the five DCs were selected as shown in column 6. This is due to the fact that the DC capacities are equal to total CPs' demands. The columns under $\widehat{V_k}$ and $\sum_{l=1}^{|L_k|} Y_{kl}$ have the same values.

The allocation decision in column under PC_j^s, shows that only two PCs, PC_1 and PC_2, have supplied to all the five selected DCs as in the case of Table 6. This also clearly shows that the existing network is not optimal. The overall total network distribution cost is $13,224,626.75 with the execution time of 23 seconds. This cost is about 15% less than the cost associated with the true capacity in Table 3, i.e., a net saving of $2.3 million. This reduction in cost is partly contributed by DC_5 which having the larger capacity than in Table 3, now serves more CPs, i.e. 29 CPs (see Table 6) as opposed to 10 in Table 3.

Table 6: Location allocation results for actual capacity in Case 1

DC_k	$\widehat{V_k}$	F_k	PC_j	S_j	Z_k	PC_j^s	$\sum_j X_{jk}$	$\lvert L_k \rvert$	$\sum_{l=1}^{Lk} Y_{kl}$
DC_1	33,190	217,229	PC_1	100,000	1	PC_1	33,190	26	33,190
DC_2	38,532	252,192	PC_2	251,000	1	PC_1	38,532	18	38,532
DC_3	24,650	161,334	PC_3	140,000	1	PC_1	24,650	13	24,650
DC_4	9,843	64,422	PC_4	41,000	1	PC_1	24,650	13	24,650
DC_5	38,929	254,790			1	PC_1 ; PC_2	3,628; 6,215	9	9,843
Total	145,144	949,967		532,000			145,144	95	145,144

Case 2: DCs' multi-capacity and computational results

In this case, the main focus is given to the use of multiple capacities per DC. Unlike Case1, here the capacity of a selected DC is an optimization decision. Hence the mathematical model used in this case is re-written as follows:

$$\underset{X_{jk}, Z_k^r, Y_{kl},}{Min} \quad \lambda[\sum_j \sum_k C_{jk} X_{jk} + \sum_k \sum_l T_{kl} Y_{kl}] + \sum_k \sum_r F_k^r Z_k^r \qquad (9)$$

subject to

$$(2), (5), (6) \& (7) \qquad (10)$$

$$\sum_k X_{jk} = \sum V_k^r Z_k^r, \ \forall k \qquad (11)$$

$$\sum_j Z^r{}_k \leq 1, \forall k \qquad (12)$$

$$\sum_l Y_{kl} \leq \sum_r V^r_k{}_k Z_k^r \forall k \qquad (13)$$

$$Z_k^r \in \{0,1\}, \forall r, k \qquad (14)$$

Where:

- Constraints (11) refer to the amount supplied from PC_j to all selected DC_k, that satisfy the DCs' capacity level V_k^r;
- Constraints (12) are now introduced to make sure that only one capacity level of selected DC is chosen. If DC_k is selected then the constraint (12) makes sure that only one of its capacity is chosen. i.e., $\sum_r Z_k^r = 1$. If DC_k is not chosen, then $\sum_r Z_k^r = 0$;
- When DC_k is selected along with a capacity level, then constraint (13) makes sure that its V_k^r for some r is not violated. The values of r ≥1, are in different ranges, some less than or equal to $\widehat{V_k}$ and some are more than $\overline{V_k}$, the existing true capacity.
- Constraints (14) are the binary values to the location variable.

The objective function (9) differs from the existing ones in literature in that the last term is modified to account for the dependence of F_k^r or V_k^r. The optimization of the model (9)–(14) uses the inputs data as in Case 1 except for each DC_k, which uses 14 different capacities, i.e. $R_k = \{V_k^1, V_k^2, \ldots\ldots, V_k^{14}\}$ has been done. In the given capacities, the Case 1 capacities, $\widehat{V_k}$ and $\overline{V_k}$ are also included. Values in the set R_k are independent of k. The set R_k, k = 1; 2,…, 5, contains the capacities that are generated randomly in [9,843:145,144]. The interval from 9,843 to 145,144 was used since this range contains the minimum capacity as observed in DCs' actual capacity $\widehat{V_k}$, and also the maximum actual capacity used.

The optimization in this case is performed with respect to all four decisions, as done in the previous model ((a) – (d)). By optimizing the model using the original five DCs the results obtained are summarized in Table 7, where the optimized capacity chosen are shown in brackets in the column under V_k^r . The results in Table 7 are self-explanatory. Table 7 shows that only four DCs are selected.

Table 7: Location allocation results in Case 2

DC_k	PC_j	S_j	$(Z_k^r;r)$	V_k^r	F_k^r	PC_j^s	$\sum_j X_{jk}$	$\lvert L_k \rvert$	$\sum_{l=1}^{\lvert L_k \rvert} Y_{kl}$
DC_1	PC_1	100,000	(1;3)	V_1^3(25,000)	F_1^3	PC_1	25,000	20	25,000
DC_2	PC_2	251,000	(0;-)	-	-	-	-	-	-
DC_3	PC_3	140,000	(1;3)	V_3^3(71,000)	F_3^3	PC_1	71,000	39	71,000
DC_4	PC_4	41,000	(1;13)	V_4^{13}(16,000)	F_4^{13}	$PC_1;PC_2$	4,000 ; 12,000	13	16,000
DC_5			(1;3)	V_5^3(33,144)	F_5^3	PC_2	33,144	24	33,144
Total		532,000		(145,144)			145,144	96	145,144

NOTE: \`-' This means the corresponding DC is not selected.

The optimal decisions for the capacity of DCs are presented in column 8 under $\sum_j X_{jk}$ As shown in Table 7, the total optimal capacity of the four selected DCs is 145,144 tons which is the same as the total CPs' demand. DC_3 has the largest capacity (71,000 tons) for all the selected DCs. This is an increase of 32,000 tons from its true capacity of 39,000 tons ($\hat{V}_k$). DC_5 also needs to be increased from its true capacity of 14,500 tons to the capacity of 33,144 tons. The results obtained indicate the need for expansions for the capacities of DC_3 and DC_5.

The overall distribution cost obtained after 16 seconds is $12,660,522.80. The total cost attained in Case 2 is the best solution for the existing maize crop distribution network in Tanzania. The cost has decreased, in comparison to Case 1, by 4.27% (actual capacity) with a net saving of $564 thousand which is an important saving to be considered. In the case of true capacity, the saving is 18.69% which is equivalent to $2,910,000. The saving is contributed by using many capacities that the program will select the best in each DC as compared to a single capacity as used in Case 1.

Conclusion and Recommendations

This study applied mathematical programming model to solve a two-level facility location problem for the Tanzania emergence maize distribution network where 93 CPs, 5 DCs and 4 PCs are considered. The total cost obtained in the computational results considered the three layers simultaneously. This is different from the existing system which is manually operated and has two different departments working independently. The existing distribution system has two different independent tasks carried out by specific different government departments. The first task is dealing with buying maize from PCs and transporting them for stocking in DCs which is done by the NFRA (first department). The second task is the transportation of maize from DCs to CPs which is done by the disaster management department in the Prime Minister's Office (PMO) (second department). This results into high costs due to fragmented co-ordination since the two departments operate independently and under different ministries which breeds inefficiency. The integrated coordination, as supported by this study, will reduce the costs and offer a more flexible system. Based on the discussed facts from optimization results, the study concludes the following: The use of optimization as a decision tool is an important aspect to be considered by the agencies in food security system and other sectors. For example, the saving of TZS 2.9 billion is a significant amount achieved through optimization. The results from this study can be applied to organize the current distribution system. In particular, Case 2 can be implemented since the existing DCs can be used together with storage facilities for the restocking of the DCs like DC_3 and DC_5. It is possible to optimize activities of food production, storage and final distribution to customers. In order to achieve this, data availability, coordinating management and the funding to the coordinating team are of great importance.

The following are recommended for further studies:

1. Extending the study to other crops especially rice which is also a common staple food. Since these products are transported to the same locations and customers, a study that involves the combination of the two products is a valuable contribution.
2. Although the study involved a mixed integer programming problem, with the exception of the binary variables, the rest were continuous variables and there possible to obtain optimal solution through available software. However, inclusion of more factors such as extension to more customer centres may require heuristic techniques.
3. The current problem considers deterministic parameters; however more realistic version considers demand as stochastic since disasters are unpredictable. Stochastic programming and other mathematical techniques may be considered as a further research area.

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