An Empirical Performance Comparison of Meta-heuristic Algorithms for School Bus Routing Problem

Sherehe Semba[1] **and Egbert Mujuni**[2]
[1]Department of Physics, Mathematics and Informatics, Dar es Salaam
University College of Education
[2]Department of Mathematics, University of Dar es Salaam
Corresponding author: emujuni@udsm.ac.tz

Abstract

School Bus Routing Problem is an NP-hard Combinatorial Optimization problem. Thus, mega-heuristic algorithms are widely used to solve instances of the School Bus Routing Problem with large data. In this work we present a model of the School Bus Routing Problem and empirical performances comparison between three meta-heuristic algorithms named Simulated Annealing (SA), Tabu Search (TS) and Ant-Colony Optimization (ACO) on the problem. We have analyzed their performances in terms of solution quality. The results show that all three algorithms have the ability to solve the School Bus Routing Problem. In addition, computational results show that TS performed best when execution time is not restricted while ACO had relative good performance when time is restricted but poor when the time is unrestricted.

Keywords: School Bus Routing Problem, Combinatorial Optimization, Meta-heuristic Algorithms

Introduction

Many of the real-life optimization problems can be formulated as Combinatorial Optimization (CO) problems. A Combinatorial Optimization problem deals with finding the best solution within a finite set of feasible solutions. Generally, a CO problem is asking to find *min* or *max* $\{f(x): x \in S\}$, where S is a finite set of a feasible solution and f is a cost function. Since S may have a huge cardinality, such problem cannot be solved by enumerating all possible solutions in a reasonable time. Due to the practical importance of CO problems, many algorithms for solving them have been developed. These algorithms can be categorized as either exact or approximate. Exact algorithms guarantee to find optimal solution for each instance of a CO problem in bounded time. Unfortunately, many Combinatorial Optimization problems arising from real life situation are NP-hard (Raghavendra 2009), so no polynomial time algorithm exists, assuming that $P \neq NP$. Therefore in the worst case, exact methods might need exponential computation time. This implies that exact methods cannot be used to solve NP-hard problems with large instances. Approximate algorithms seek to find solutions which are as close as possible to the optimum values within reasonable amount of time.

Among the widely used methods in solving NP-hard problems are heuristic algorithms, which give quick and good solutions without guaranteeing that the solutions obtained are optimal. Heuristic algorithms are examples of approximate algorithms. The development of heuristic algorithms leads to meta-heuristic algorithms. In this paper we explore the application of meta-heuristic algorithms on the well-known hard CO problem called School Bus Routing

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Problem (SBRP). In particular, we compare performance of three heuristics on the SBRP.

The remaining part of this paper is organized as follows: The next section gives a brief literature review on the SBRP. We also present our mathematical model of the SBRP. In another section meta-heuristic algorithms are described and few previous studies of the meta-heuristics comparisons are presented. In subsequent sections, the experiments are described and the results are presented and discussed. Conclusions and future research directions are provided in before the references.

Literature Review

The School Bus Routing (SBRP) falls into a larger class of routing and scheduling problem, called the vehicle routing problem (VRP). The VRP is a combinatorial optimization problem which can be specified as follows: a set of vehicles provide service to a group of spatially distributed customers. The problem is to find a set of vehicle routes and schedules that satisfies a variety of constraints and minimize the total fleet operating cost. The SBRP involves the transportation of students from home to school in the morning and from school to home in the evening.

Introduced by Newton and Thomas (1969), School bus routing and scheduling has become an area where operations research has much success. Braca et al. (1997) reported that many communities around the world have implemented and used computerized routing systems that in most cases lead to reduction in operating costs. It is thus not surprising that the SBRP has received considerable attention among researchers. A list of works on SBRP includes: Bowerman et al. (1995), Corbe`ran et al. (2000), Li and Fu (2002), Schittekat et al. (2006), Bektas and Elmastas (2007), Arias-Rojas et al. (2012), Kim and Park (2013), Ngonyani et al. (2015), Schittekat et al. (2013) and Manumbu et al. (2014). A

comprehensive survey can be found in Li and Fu (2002) and Park and Kim (2010).

The SBRP is known to be NP-hard (see. e.g., Fugenschuh 2009). This NP-hardness implies that it is very unlikely to solve it in polynomial time. Thus, a large number of researchers have focused on finding heuristic algorithms to solve the problem. Corbe`ran et al. (2000) used Scatter Search heuristic to address the SBRP in a rural area with the desire of minimizing the number of buses used to transport students from their homes to school and back. Arias-Rojas et al. (2012) formulated the SBRP as a classical capacitated Vehicle Routing Problem, and solved the resulting model using Ant-Colony heuristic. The results showed that the proposed approach found a reduction of 15.2% of the total cost of student picked up to go to school, and then delivered back to their home. It reduces students travel time and hence improving their quality of life. Manumbu et al. (2014) formulated a mathematical model with the aim of minimizing the amount of time spent by the students in the buses from the point where they are picked up to the school. They used Simulated Annealing (SA) heuristic to solve the resulting model. Recently, Ngonyani et al. (2015) presented mathematical model for the School Bus Routing Problem with the objective of minimizing the total time used by the students to travel to and from the school. They used TS heuristic to find optimal routes. When the model was applied to real data from a school in Dar es Salaam, it was found that the total travel time spent by the students in the buses could be reduced by 19.33%.

Many algorithms have been presented but on different sets of data. This paper compares performances of three heuristic methods on the same data set to give an insight into their performances. This will contribute to the understanding of which of these three heuristics may be more effective in solving different problems.

Mathematical Model

One of the assumptions in the Ngonyani et al. (2015) model is that bus-stops are linearly ordered. Manumbu et al. (2014) formulated a model without making this assumption. However, one of the assumptions in Manumbu et al. (2014) is that each pick up point is served by only one bus. A mathematical model in this work is developed under similar assumptions to those made in Manumbu et al. (2014), an exception is that we are not assuming that each pick up point is served by one bus. The following sets, parameters and variables are used.

Sets:

$Q = \{1, 2, 3, ..., K\}$ is a set of the available buses to be used where K is the total number of available buses.

$P = \{1, 2, 3, ..., S\}$ is a set of all pick up points where one or more students are picked up where by S is a total number of stops arranged scattered around the school and $S + 1$ denotes the school.

Parameters:

C_k = Capacity of bus $k \in Q$

V_i = Number of students at stop $i \in P$

T_{ij} = Travel time from stop $i \in P$ to stop $j \in P$

β = Average pick up time of one student.

Variables:

Z_i = Set of buses visited stop $i \in P$.

Y_k = Set of stops to be visited by bus $k \in Q$.

P_{ki} = The ith stop to be visited by bus k.

$X_{P_{ki}}$ = Number of students picked up by bus k at stop $P_{ki} \in P$.

The objective of the model presented is to minimize the total time spent by the students to travel to and from the school by varying buses routes.

Minimize

$$f = \sum_{k=1}^{K} \left\{ \sum_{i=1}^{|Y_k|} \left(T_{P_{ki}, P_{k(i+1)}} \left(\sum_{l=1}^{i} X_{P_{kl}} \right) + \beta X_{ki} \left(1 + \sum_{l=1}^{i-1} X_{P_{kl}} \right) \right) \right\}$$

Subject to:

$$1. \sum_{i=1}^{|Y_k|} X_{P_{ki}} \leq C_k, k = 1, 2, 3, ..., K$$

$$2. P_{k(|Y_k|+1)} = S + 1, k = 1, 2, 3..., K$$

$$3. \sum_{k \in Z_i} X_{P_{ki}} = V_i, i = 1, 2, ..., S$$

$$4. X_{P_{ki}} \geq 0, \text{ and an integer.}$$

Where,

$\sum_{k=1}^{K} \left\{ \sum_{i=1}^{|Y_k|} \left(T_{P_{ki}, P_{k(i+1)}} \left(\sum_{l=1}^{i} X_{P_{kl}} \right) \right) \right\}$ is the total travelling time spent by the students within the bus in all stops by all buses and $\sum_{k=1}^{K} \left\{ \sum_{i=1}^{|Y_k|} \left(\beta X_{ki} \left(1 + \sum_{l=1}^{i-1} X_{P_{kl}} \right) \right) \right\}$ is the total pick up time of students at all bus stops.

Constraint (1) ensures that the sum of students picked up in all stops, must not exceed the bus capacity, constraint (2) ensures that all buses finish their routes at a school, constraint (3) ensures that all students at stop i are assigned to some school buses, and constraint (4) ensures that the number of students assigned to each bus at each bus stop is nonnegative.

Note that our model has the same objective function as the model of Manumbu et al (2014). The main difference between these two models is that our model has an additional constraint (3) which ensures that all students at any bus stop are picked up. This was necessary because, in our model, we are not assuming that each bus stop is served by only one bus.

Meta-Heuristic Algorithms

Meta-heuristic algorithm is a higher-level procedure or heuristic that utilizes an interaction between local improvement procedures and upper level strategies that creates a process for escaping from getting

the local optimal solution and performing a robust search of a solution space. Thus, meta-heuristic algorithms are heuristic algorithms with the powerful mechanism to archive a better solution. Hence, the solutions obtained when solving the problem using meta-heuristic algorithms usually have better quality compared to the solutions obtained when using basic heuristic algorithms.

Meta-heuristic algorithms have been successfully used to solve a number of real-life problems. Examples include Traveling Salesman Problem (Mladenovic and Hansen 1997, Reinelt 1994), Timetabling Problem (Burke et al. 2007, Mushi 2011), Job Scheduling Problem (Sayadi et al. 2010, Ruiz and Vazquez-Rodriquez 2010), and Vehicle Routing Problem (Nguyen 2014).

Major components of meta-heuristic algorithms are diversification and intensification. Diversification is the ability to explore many and different regions of the search space, while intensification is the ability to obtain high quality solution within the explored regions (Lozano and Martinez 2010). Diversification and intensification stem from the Tabu Search (Glover and Laguna 1997). In the evolutionary computation field instead of diversification and intensification the terms exploration and exploitation are used. The notions of exploitation and exploration refer to short term strategies tied to randomness, while intensification and diversification refer to the medium and long term strategies based on the usage of memory.

The efficiency of meta-heuristic algorithms depends main on two things: first is the capability of generating the new solutions that can usually be more likely to improve the existing solutions and also to cover most important search areas where the global optimum may lie. Second is the capability of escaping being trapped into local solutions. Examples of meta-heuristic algorithms include Ant Colony Optimization (ACO), Genetic Algorithms (GA), Iterated Local Search (ILS), Simulated Annealing (SA), Particle Swarm (PS), Firefly Algorithm (FA), Harmony Search (HS), Cuckoo Search (CS), Honeybee Algorithm, Scatter Search, Tabu Search (TS) and Great Deluge Algorithm (GDA).

In this study we have chosen SA, TS and ACO to solve the model presented. Below are detailed descriptions of these algorithms.

Simulated Annealing

Simulated Annealing is a probabilistic meta-heuristic algorithm. It has been devised, so as to avoid being trapped into poor local optima by accepting bad moves according to a probability function. The method imitates the annealing process in metallurgy; starting from a randomly generated solution, a neighboring solution is compared with the current solution according to an appropriate probability function. The acceptance and rejection of the bad move is restricted by a probability function. A pseudo code is given in Figure 1.
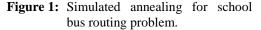
Many studies, including Dowsland (1995) and Kouvelis and Chianga (1992) have presented different ways of selecting the initial value of temperature. In this study, initial temperature is chosen such that it can capture the entire solution space. We choose a very high initial temperature as it increases the solution space. However, at a high initial temperature, Simulated Annealing performs a large number of iterations, which may be giving better results. Therefore, the initial temperature chosen in this experimentation is 100.

It is known from literature that the performance of the Simulated Annealing algorithm depends strongly on the chosen cooling schedule. A number of cooling schedules have been proposed by different authors. A list of such schedules includes logarithmic, exponential cooling, geometric and linear cooling (see, e.g., Aarts et al. 1988, Azencott 1992, Mushi 2011). In this work we have applied the geometric function which is given by $f(T) = \omega T$, where the cooling rate

ω is a constant value between 0.8 and 0.99. The choice of geometric cooling scheme was made due to the fact that is one of the most widely used schemes (see, e.g., Mushi 2011) and it is simple to implement.

```
START
INPUTS: Initial solution S_0; Initial
Temp T_0; Freezing F; Cooling Rate = ω;
k = 0;
S_best = S_0;
WHILE (k < MaxIteration)
  T = T_0 ; S' = S_0; k = k + 1;
  WHILE (T > F)
  Generate solution S'' from S' as
follow:
  Randomly choose bus route B_1
  Randomly choose a bus stop s' from
B_1
  Randomly choose bus route B_2
    Move s' from B_1 to B_2
  Compute Δ = f(S'') − f(S')
  IF (Δ < 0)
      S' = S''
  ELSE
    Generate random value  μ ∈ (0,1)
  IF (e^{Δ/T} > μ);
      S' = S''
  END IF
  END IF
  T = ωT
END WHILE
  IF f(S') ≤ f(S_Best)
    S_Best = S'
 ENDIF
END WHILE
  RETURN S_best as best solution;
  END
```

Figure 1: Simulated annealing for school bus routing problem.

Tabu Search

Tabu Search is a neighborhood search method which employs "intelligent" search and flexible memory technique to avoid being trapped at local optimal solution. Both short term and long term memories are used,

in order to improve the exploration quality. The long-term memory is used to diversify the search into new regions. To avoid visiting the same solution during the iteration, the recently selected solution is pushed into the tabu list so that it becomes a "taboo" for a specified period. The best neighbor of the current solution $N(x_c)$ is chosen. A pseudo code is given in Figure 2.

```
START
  Get Initial Solution S_0
  Set S = S_0; S_best = S; k = 0; TL = ∅;
  WHILE (k<MaxIteration) Do
    Generate set V ⊆ N(S)
    Choose a candidate solution  S' ∈
    (V − TL) such that f(S') ≤
  f(S'') ∀ S'' ∈ V
    IF f(S') ≤ f(S_Best)
      S_Best = S'
    ENDIF
    Push S' into TL
    Set S = S'
    Update TL
    k = k+1
  END WHILE
  Return S_Best as the best solution
  END
```

Figure 2: Tabu search for school bus routing problem.

Tabu list is used to store some attributes of recently visited solutions with the aim of discouraging the search from going back to recently visited solution. This prevents the occurrence for certain period called tabu tenure. The tabu tenure is an important factor to guide the search. It influences the performance of the method. Tabu tenure can be static or dynamic depending on the type of the problem and the size of the instance. In static tabu tenure, its value is fixed throughout the search while dynamic tabu tenure varies during the search. Initially the tabu list is set empty. Salhi (2002) provides different ways for defining tabu tenure. This includes fixing a predetermined value,

randomly choosing from a specific range, or dynamically changing by adjusting the value. In this work, a predetermined fixed value was used. It is known from literatures that experiments of varying tabu tenure should be done to choose the best tabu. The work of Ngonyani et al. (2015) on SBRP indicates that tabu tenures with short values give good results as compared to those of large values. Thus, in this work the values tested were 3, 5 and 7.

Ant colony algorithm

Ant Colony Optimization (ACO) algorithm, introduced in Dorigo et al. (1991), is a probabilistic mega-heuristics that imitates cooperative behavior of ants in finding food for their colonies. Ants are able to find a shortest path between a food source and the nest by using a trail system, called pheromone. Ants start searching for food by walking randomly in the area near to their nest. While moving, ants release a pheromone trail on the ground. When they choose their way, they choose with higher probability paths that are marked by stronger pheromone concentrations.

With ACO, the optimization process involves a group of K ants where by each ant builds a solution to the problem. Each move is based on two ingredients: trails and attractiveness, which are respectively denoted by τ_{ij} and η_{ij}. In addition, the optimization process includes two processes; namely, trail evaporation that reduces all trail values over time in order to avoid any possibility of being trapped into local optimal and daemon actions that can be used to bias the search process from non-local perspective.

The algorithm begins by initializing the attractiveness η_{ij} of the move which is computed by some heuristics indicating a prior desirability of that move and the trail level τ_{ij} of the move indicating a posterior desirability of that move.

The phases of Ant Colony algorithm are; First is an initialization phase in which an initial value of τ_0 is given to τ-values and each artificial ant (bus) k is assigned to a randomly chosen bus stop.

The probability of that stop j is selected by ant k after visiting stop i is calculated as follows

$$P_{ij}^k = \begin{cases} \dfrac{(\tau_{ij})^\alpha (\eta_{ij})^\beta}{\sum_{s \in X_k} (\tau_{is})^\alpha (\eta_{is})^\beta} & \text{if } j \in X_k \\ 0 & \text{otherwise} \end{cases}$$

(Dorigo et al. (1991)) where τ_{ij} and η_{ij} are the trail level and attractiveness, respectively, between stops i and j; α and β parameters that control the balance between the influence of the trail and attractiveness, and X_k is the set of next possible stops from the current bus stop. We always put $\eta_{ij} = 1/T_{ij}$. At the end of each cycle, the values τ_{ij} are updated as follows:

$$\tau_{ij}(t+1) = \rho(\tau_{ij}) + \Delta\tau_{ij}$$
$$\Delta\tau_{ij} = \sum_{k=1}^{P} \Delta\tau_{ij}^k$$
$$\Delta\tau_{ij}^k =$$
$$\begin{cases} \gamma/f & \text{if ant } k \text{ makes a move } (i,j) \\ 0 & \text{otherwise} \end{cases}$$

A pseudo code is given in Figure 3.

```
START
Initialize: μij and τij ∀i,j; k=0
WHILE (k<MaxIterations)
  FOR each ant k (currently in stop i)
    Choose in probability the stop to move into
    Push the chosen stop to the kth ant's set
      Tabuk
    Until ant k has completed its solution
    Compute Δτij = Σm_{k=1} Δτ^k_ij
    Update the local trail matrix:
  END FOR
    τij(t) = ρτij(t-1) + Δτij ∀i,j
END WHILE
END
```

Figure 3: Ant colony algorithm for school bus routing problem.

Mega-Heuristics Comparisons

Because of the NP-hardness of many combinatorial optimization problems, a number of heuristics algorithms have been developed. Literature shows that heuristics perform well in some problems but perform poorly in other problems. That is, all meta-heuristics usually encounter problems on which they perform poorly (see, e.g., Adewole et al. 2012). Thus, it is important for users to have experience on which heuristics work well in different classes of problems. This necessitates comparative studies on heuristic methods for various problems. The idea is to identify which methods work better for a given problem. Thus, many researchers have performed comparative studies between different meta-heuristic algorithms. In this section we give a highlight of such studies.

Azimi (2004) used Genetic Algorithm, Simulated Annealing, Ant Colony System and Tabu Search in solving the Examination Timetabling Problem and compared their results. All the algorithms used the same direct representation and were implemented in the basic components in a straightforward manner using a common search landscape for a fair and meaningful analysis. The results showed that Ant Colony Optimization and followed by Tabu Search worked better when compared to others.

Arostegui et al. (2006) did the relative performances comparison of Simulated Annealing, Tabu Search and Genetic Algorithm on various types of facility location problem under time-limited situation, without restricted conditions and with limited solutions. Tabu Search performed well in most cases. The performances of Genetic and Simulated Annealing algorithms were more partial to problem type and criterion used. In general they concluded that Tabu Search gave better results than others, and it is easy to develop and implement.

Paul (2010) compared experimental performances of Simulated Annealing and Tabu Search heuristics for solving quadratic assignment problem. The comparisons were based on various values of targeted solution quality. The results showed that for a number of varied problem instances, Simulated Annealing performs better for higher quality targets while Tabu Search performs better for lower quality targets.

Silberholz and Golden (2010) compared meta-heuristic algorithms in terms of both solution quality and run-time. It was observed that, expanding the practice of creating geometric problem instances with easy-to-visualize optimal or near-optimal solutions increases understanding of how meta-heuristic algorithms perform in a global optimization sense. They concluded that good techniques in solution quality and run-time comparisons produce the most meaningful and unbiased possible results.

Bajeh and Abolarinwa (2011) compared Genetic Algorithm and Tabu Search results for solving Scheduling Problem. The results showed that Tabu Search can produce better results, with minimal computing time, than those generated by Genetic Algorithm. However, Genetic Algorithm can produce several different good solutions at the same time because they hold the whole penetration of chromosomes which may not originated from the same parents.

Adewole et al. (2012) compared the performance of Simulated Annealing and Genetic Algorithm for solving Traveling Salesman Problem. Their results show that, Simulated Annealing runs faster than Genetic Algorithm; Genetic Algorithm shows exponential increases in execution time with the increases of the number of cities. However, in terms of solution quality Genetic Algorithm was shown to be better than Simulated Annealing.

Said et al. (2014) presented a comparative study between meta-heuristic algorithms; Tabu Search, Simulated Annealing and Genetic Algorithm for solving quadratic assignment problem, and analyzed the performances in terms of both run-time

efficiency and solution quality. The results showed that Genetic Algorithm produces better solution while Tabu Search executes faster in comparison with other meta-heuristic algorithms for solving quadratic assignment problem.

To the best of knowledge of the authors, there are no studies that have been done to compare performances of meta-heuristic algorithms for School Bus Routing Problem. This motivated this work in which we present the empirical performances comparison of three meta-heuristic algorithms (Simulated Annealing, Tabu Search and Ant Colony) that have been used to solve the School Bus Routing Problem.

Experiments and Results

Experimental results were run on a Laptop with the following configurations: CPU 1.3 GHZ, 2.0 GB RAM, Windows 7. This test was conducted by solving the school bus routing model presented above by using Simulated Annealing, Tabu Search and Ant Colony algorithms. Comparison of the algorithms is based on solution quality and execution time for real life School Bus Routing Problem.

Data used in this experiment were collected by Manumbu in 2014 from three different private schools in Dar es Salaam city, Tanzania. The schools are Atlas primary school, African Nursery & Primary School and Yemeni DYCCC Secondary School. The size of the input data is given in Table 1.

Table 1: Size of input data

Schools	Buses	Stops	Students
Atlas PS	9	68	445
African N & PS	7	65	197
Yemeni SS	5	39	113

It is known from literatures that the performance of heuristics depends on the parameter settings. Thus, we run each heuristic with data from each school a number of times using different parameters in order to identify parameters settings with best performance. Table 2 summarizes the findings.

We considered two cases. In the first case we restricted time for running heuristics, while in the second case we allowed the heuristics run and complete according to their parameters.

Table 2: Parameters with best performance

School	Simulated Anneal	Tabu Search	Ant-Colony
Yemen SS	$\omega = 0.99$ $T_0 = 100$ $F = 0.001$	Tabu size: 5	$\rho = 0.5$
African N & PS	$\omega = 0.99$ $T_0 = 100$ $F = 0.001$	Tabu size: 7	$\rho = 0.5$
Atlas PS	$\omega = 0.99$ $T_0 = 100$ $F = 0.001$	Tabu size: 7	$\rho = 0.5$

Results for restricted time

For this case, all of the three heuristics were allowed to run for a maximum time. Figures 4, 5 and 6 give comparisons of the three heuristics using data from the above mentioned schools.

For data from Atlas PS, all of the three heuristics were allowed a maximum time of 1000 seconds. Figure 4 indicates that SA had good performance during the first 300 seconds. ACO had consistent improvement on the quality of solution as the computational time increases, and it overtook SA after 300 seconds. At the end of 1000 seconds, ACO had the best performance on Atlas while TS had worst performance when compared to the other two heuristics.
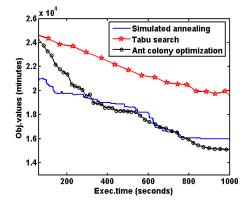
Figure 4: Results for data from Atlas Primary School.

For the case of African N & PS, all three heuristics were allowed to run for 1000 seconds. Both SA and ACO had good performance during the first 400 seconds; see Figure 5. The figure indicates that although TS had poor values of the objective function at the beginning, it continued improving its solutions. After 1000 seconds, TS had best performance, followed by SA.
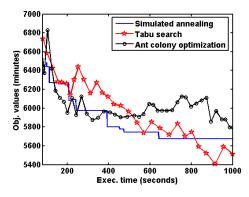

Figure 5: Results for data from African Nursery & Primary Schools.

Figure 6 gives a comparison using data from Yemen SS. For this case maximum running time was set to 800 seconds. The figure shows all three heuristics had almost similar performance. ACO produced the best solution followed by SA.
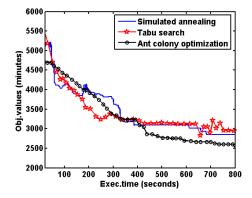

Figure 6: Results for data from Yemeni Secondary School.

Results for unrestricted time

All three heuristics were also allowed to run and finish according to their parameters, without restricting time of running. For each heuristic we calculated average time a student spends in a bus. Table 3 gives the findings. In this case TS performed best for Atlas PS and Africana N & PS. SA performed best for Yemen SS.

Table 3: Computed average time a student spends in a bus

School	Number of Students	Average time a student spends in a bus (Minutes)		
		SA	TS	ACO
Atlas PS	445	34.7	32.4	33.6
African N & PS	197	28.7	27.4	28.5
Yemen SS	113	19.9	20.9	21.2

Conclusion and Future Research Directions

In this paper, we used three mega-heuristic algorithms, Simulated Annealing, Tabu Search and Ant-Colony Optimization, to solve the real-life problem (the School Bus Routing Problem). Heuristics were tested using secondary data from three schools in Dar es Salaam, Tanzania; Atlas Primary School, African Nursery & Primary School and Yemeni Secondary School. This work was dedicated to compare performances of

these three heuristics in relation to quality of their solutions. We considered two cases: (1) when the time of running each heuristic is restricted, and (2) when the time is unrestricted.

For the case of time restricted, ACO performed best for Atlas PS and Yemen SS, while TS gave the best performance for Africa N & PS. In all the three schools, SA was ranked second. For the case of unrestricted time, TS performed best for Atlas PS and African N & PS while SA had the best performance for Yemen SS. These results show that TS performed poorly when time is restricted and performed well when time is not restricted. On the other hand, ACO had relative good performance when time is restricted but poor when time is unrestricted.

As it has been mentioned above, the performance of heuristics depends on the parameter settings. Thus, we would like to remark that the parameters of the three heuristics used may have affected the results. In addition, the selection of maximum time for a heuristic to run may have also affected the results. Thus, conclusions of this work may have been affected by these two facts. However, since this is – to the best of our knowledge- so far the only study comparing the performances of meta-heuristic algorithms on School Bus Routing Problem; it has significant contribution in the field of combinatorial optimizations, in particular in solving the School Bus Routing Problem.

There are different mathematical models for SBRP with different objectives and constraints (see, for example, Li and Fu (2002)). In this work we developed a mathematical model without time window constraints. Therefore possible future work is to extend the model to accommodate additional constraints such as time windows and lower and upper limits on the number of students per bus.

References

Aarts EH, Korst JH and van Laarhoven PJ, 1988 A quantitative analysis of the simulated annealing algorithm: A case study for the traveling salesman problem. *J. Stat. Phys.* 50(1-2): 187-206.

Adewole AP, Otubamowo K and Egunjobi TO 2012 A comparative study of simulated annealing and genetic algorithm for solving the traveling salesman problem. *Int. J. Appl. Inf. Syst. (IJAIS)* 4(4): 6-12.

Arias-Rojas JS, Jim´enez, JF and Montoya-Torres JR 2012 Solving of school bus routing problem by ant colony optimization. *Revista EIA* 17: 193-208.

Arostegui Jr MA, Kadipasaoglu SN and Khumawala BM 2006 An empirical comparison of tabu search, simulated annealing, and genetic algorithms for facilities location problems. *International Journal of Production Economics* 103(2): 742-754.

Azencott R 1992 Sequential simulated annealing: speed of convergence and acceleration techniques. In Azencott, ed: *Simulated Annealing: Parallelization Techniques*, Wiley, New York, 1-10.

Azimi ZN 2004 Comparison of metaheuristic algorithms for examination timetabling problem. *Journal of Applied Mathematics and Computing* 16(1-2): 337-354.

Bajeh AO and Abolarinwa KO 2011 Optimization: A comparative study of genetic and tabu search algorithms. *Int. J. Comp. Appl.* 31(5): 43-48.

Bektas T and Elmastas S 2007 Solving school bus routing problems through Integer programming. *Journal of the Operational Research Society* 58(12): 1599-1604.

Bowerman R, Hall B and Calamai P 1995 A multi-objective optimization approach to urban school bus routing: Formulation and solution method. *Transp. Res. Part A: Policy Practice* 29: 107-123.

Braca J, Bramel J, Posner B and Simchi-Levi D 1997 A computerized approach to the

New York City school bus routing problem. *IIE Transactions* 29(8): 693-702.

Burke KE, McCollum B, Meisels A, Petrovic S and Rong Qu 2007 A graph-based hyper-heuristic for educational timetabling problem. *Eur. J. Oper. Res.* 176(1):177-192.

Corbe`ran A, Fernandez, E, Laguna M and Marti R 2002 Heuristic solution to the problem of routing school buses with multiple objectives.\ *J. Oper. Res. Soc.* 53(4): 427-435.

Dorigo M, Maniezzo V, and Colorni A 1991 The Ant System: An Autocatalytic Optimizing Process, *Technical Report TR91-016*, Politecnico di Milano.

Dowsland KA 1995 Simulated annealing. In: Reeves CR (Ed) *Modern Heuristic Techniques for Combinatorial Problems*, McGraw-Hill, 20-69.

Fügenschuh A 2009 Solving a school bus scheduling problem with integer programming. *Eur. J. Oper. Res.* 193 (3): 867-884.

Glover F and Laguna M 1997 Tabu Search: Kluwer Academic Publishers, Boston.

Kim T and Park BJ 2013 Model and algorithm for solving school bus problem. *J. Emerg. Trends Comp. Inf. Sci.* 4(8): 596- 600.

Kouvelis P and Chiang W 1992 A simulated annealing procedure for single row layout problem in flexible manufacturing systems. *International Journal of Production Research* 30: 717-732.

Li L and Fu Z 2002 The school bus routing problem: A case study. *J. Oper. Res. Soc.* 53: 552-558.

Lozano M and Martinez GC 2010 Hybrid meta-heuristics with evolutionary algorithms specializing in intensification and diversification: Overview and progress report. *Computers and Operations Research* 37: 481-497.

Manumbu DM, Mujuni E and Kuznetsov D 2014 Mathematical formulation model for a school bus routing problem with small

instance data. *Math. Theory Model.* 4(8): 121-132.

Mladenovic N and Hansen P 1997 Variable neighborhood search. *Comp. Oper. Res.* 24(11): 1097-1100.

Mushi AR 2011 Two phase heuristic algorithm for the university course timetabling problem: the case of University of Dar es Salaam. *Tanzania Journal of Science* 37: 73-83.

Newton RM and Thomas W 1969 Design of school bus routes by computer. *Soc.- Econ. Plan. Sci.* 3(1): 75-85.

Ngonyani B, Mujuni E and Mushi A 2015 Optimizing schedules for school bus routing problem: the case of Dar es Salaam schools. *Int. J. Adv. Res. Comp. Sci.* 6(1): 132-136.

Nguyen KP 2014 *Meta-heuristics solution methods for rich vehicle routing problem.* PhD thesis, Inter-university Research Center on Enterprise Networks, Logistics and Transportation.

Park J and Kim BI 2010 The school bus routing problem: A review. *Eur. J. Oper. Res.* 202(2): 311-319.

Paul G 2010 Comparative performance of tabu search and simulated annealing heuristic for quadratic assignment problem. *Oper. Res. Lett.* 38: 577-581.

Raghavendra P 2009 *Approximating NP-hard problems efficient algorithms and their limits.* PhD Dissertation, University of Washington.

Reinelt G 1994 The Traveling Salesman: Computational Solutions for TSP Applications, Springer-Verlag, Berlin.

Ruiz R and Vázquez-Rodríguez AJ 2010 The hybrid flow shop scheduling problem. *Eur. J. Oper. Res.* 205: 1-18.

Said GA, Mahmoud AM and El-Horbay EM 2014 A Comparative study of meta-heuristic algorithms for solving quadratic assignment problem. *Int. J. Adv. Comp. Sci. Appl.* 5(1).

Sayadi M, Ramezanian R and Ghaffari-Nasab N 2010 A discrete firefly meta-heuristic with local search for makespan

minimization in permutation flow shop scheduling problems. *Int. J. Indust. Eng. Comput.* 1(1): 1-10.

Salhi S 2002 Defining tabu list size and aspiration criterion within tabu search methods. *Comput. Oper. Res.* 29: 67-86.

Schittekat P, Sevaux M and Sorensen K 2006 A mathematical formulation for a school bus routing problem. In IEEE *Proceedings of the 2006 International Conference on Service Systems and Service Management*: 1552-1557.

Schittekat P, Kinable J, Sörensen K, Sevaux M, Spieksma F and Sringael J 2013 A Meta heuristic for the school bus routing problem with bus stop selection. *Eur. J. Oper. Res.* 229(2): 518-528.

Silberholz J and Golden B 2010 Comparison of metaheuristics. In: Gendreau M and Potvin JY (Eds) *Handbook of Metaheuristics. International Series in Operations Research & Management Science*, vol 146 (pp. 625-640). Springer, Boston, MA.