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FAHP-TOPSIS model in ranking critical success factors of e-learning: A comparative analysis of normalization methods

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Keywords

Critical Success Factors;
Fuzzy Analytic Hierarchy Process;
Technique for Order Preference by Similarity to Ideal Solution;
Vector Normalization

Abstract

The teaching and learning process has become effective and explicit due to the progressive development of e-learning. This has significantly transformed the education framework. As a result, evaluating the usefulness of e-learning requires a detailed understanding of the critical success factors that impact the success of e-learning environments. This study ranked these factors using Multi-Criteria Decision-Making techniques. Specifically, the Fuzzy Analytical Hierarchy Process was used to determine the weight of each factor, while the Technique for Order of Preference by Similarity to Ideal Solution was used to rank the factors of e-learning. The study also examined and compared four normalization methods commonly applicable in the Technique for Order of Preference by Similarity to Ideal Solution. The results showed Vector, Max and Sum normalization methods produced highly consistent rankings which differ from the ranking produced by the Max-Min method. This finding is significant as it indicates the choice of normalization technique can influence the final ranking of factors.

Introduction

Information and Communication Technology (ICT) promotes global economic and social development (Twaakyondo 2012). Countries are embracing ICT in fields such as education (IIEP 2023) to enhance teaching and learning outcomes while addressing the digital divide. e-learning has grown in developed and developing nations, with developed countries showing greater integration (Feghali et al. 2006, Kilewo and Rwabishugi 2021). In Tanzania, stakeholders have promoted and implemented e-learning systems, with UNESCO and HDIF funding projects like TIE-online library (Mtebe and Raphael 2018). Other e-learning projects include Vodacom Tanzania Foundation e-Faraja Initiative, Maktaba Tetea, Shule Direct, Ubongo Kids, Mtabe, and Smart Class. Understanding effective e-learning factors is crucial for managers and decision-makers (Roffe 2002, Tzeng et al. 2007). Multi-Criteria Decision Methods (MCDM) can optimize decision-making for e-learning by evaluating and prioritizing multiple critical success factors.

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MCDM is a systematic approach to decision-making in operational research, involving mathematical and computational tools for evaluating multiple alternatives (Behzadian et al. 2012, Guo and Zhao 2017). It supports decisions in various contexts. Studies have ranked e-learning Critical Success Factors (CSFs) using MCDM approaches.

Despite considerable research on e-learning and various MCDM approaches evaluating its effectiveness (Krotov 2015, Zare et al. 2016), studies addressing e-learning implementation for Tanzanian secondary schools are lacking. This study aimed to facilitate e-learning in Mathematics by analyzing CSFs using the FAHP-TOPSIS ranking model and comparing TOPSIS normalization methods. Normalization in MCDM transforms performance ratings into a computable format for comparing alternatives (Chakraborty and Yeh 2009). Effective normalization enables MCDM methods to evaluate alternatives (Vafaei et al. 2016). To ensure compatibility across scales, a single normalization method is often used in MCDM. This study examines four normalization methods: Vector normalization scales data based on Eucledian length of the vector, linear scale transformation (Max-Min) adjusts values to a fixed range based on maximum and minimum values, linear scale transformation (Max) divides each value by the maximum in its column, while linear scale transformation (Sum) scales each value relative to the sum of all values in the column. Studies on the impact of normalization methods in MCDM problems indicate that certain methods are more suitable for specific decision methods. Chakraborty and Yeh (2009) evaluated these normalization techniques within the MCDM simple additive weight (SAW) method, concluding that vector normalization is optimal for SAW and TOPSIS. Vafaei et al. (2016) confirmed these findings using Pearson and Spearman correlation coefficients, identifying vector normalization as the most effective for TOPSIS.

Most studies have focused on higher education institutions without employing advanced decision-making techniques to capture the complexity of e-learning factors (Lwoga 2014, Mtebe and Raphael 2018). Many used basic statistics or single MCDM techniques like AHP and TOPSIS, which inadequately address the uncertainty of real-world decision-making. In Tanzania, few studies have applied MCDM methods. For example, Kadenge et al. (2019) focused on water sector where water loss management strategies were optimized while Mng'ong'o et al. (2022) applied Fuzzy TOPSIS in agriculture to assess performance of agro-processed crops. Neither study addressed education-related applications, particularly evaluating the effectiveness of e-learning in secondary schools. Moreover, these studies disregarded the effect of normalization methods on MCDM ranking results, a methodological gap the current study aims to fill. This study adopts a combined FAHP and TOPSIS method, enabling effective evaluation through decision-making tools under uncertainty.

The FAHP-TOPSIS model has been used for selecting alternatives in various domains such as manufacturing (Sequeira et al. 2022), business (Zavadskas et al. 2018, Kustiyahningsih et al. 2022, Rodríguez et al. 2023), information security (Mohyeddin and Gharaee 2014), operational efficiency (Wang et al. 2023), power plant management (Mousavi et al. 2022), healthcare, and data analysis (Alharbi et al. 2024). However, applications remain limited in educational settings (Mardani et al. 2015), particularly in e-learning (Başaran and Haruna 2017, Volarić et al. 2014).

Previous studies have focused on the evaluation of specific tools rather than broader e-learning success factors. Başaran and Haruna (2017) evaluated mobile learning applications for mathematics, focusing on features and usability, while Volarić et al. (2014) used FAHP and TOPSIS to select multimedia applications for teaching. Both narrow their focus to tool evaluation rather than broader factors affecting e-learning success. This study shifts focus to identifying and prioritizing critical success factors (CSFs) for e-learning, thus addressing the broader systemic factors influencing e-learning effectiveness in secondary education.

Moreover, this study contributes to the methodological discourse by comparatively analyzing the normalization methods that impact the reliability and validity of rankings, offering a new insight that extends current knowledge.

To address the mentioned research gaps, this study aims to check the validity of the following hypotheses.

i. The most critical success factors influencing the effective implementation of e-learning in secondary schools in Tanzania exist.
ii. The relative closeness coefficient of each CSF of e-learning is consistent and reliable across all selected normalization methods for the FAHP-TOPSIS model.
iii. The choice of a normalization method significantly impacts the ranking of CSFs of e-learning for the FAHP-TOPSIS model.
iv. The comparative analysis of normalization methods holds significant implications for the application of MCDM in various fields. The purpose of this study is, therefore, to enhance the effective implementation of e-learning in mathematics for Tanzania secondary schools by identifying and ranking critical success factors, and to evaluate how different normalization techniques influence the consistency and reliability of the ranking outcomes.

Material and methods

This research employs the FAHP-TOPSIS model which is suitable for complex decision problems, enabling integration of subjective and objective measures. The study adopted CSFs from existing e-learning research, evaluated through expert inputs from Mathematics teachers and IT specialists, in five dimensions. Questionnaires were developed for Fuzzy AHP and TOPSIS approaches. The survey was conducted in Dar es Salaam, which has 350 secondary schools, the highest number in Tanzania according to the 2022 National Census. The region's high population density, diverse socio-economic backgrounds, and better access to e-learning resources make it a representative sample for informing national e-learning strategies.

FAH-TOPSIS model

The primary goal of this study was to establish a framework that evaluates the critical success factors of e-learning incorporating MCDM methods. The following are the step-by-step procedures used by this study to develop the FAHP-TOPSIS model for ranking CSFs of e-learning;

Identification of e-learning CSFs

The study conducted a literature review and expert consultations to identify the CSFs for e-learning in secondary schools. The twenty-five CSFs categorized in five dimensions were formulated. Table 1 shows the dimensions with their corresponding CSFs obtained;

The FAHP model

The study employed FAHP to determine the weights of the identified CSFs. With the assistance of experts judgments pairwise comparison matrices were constructed. The fuzzy logic was incorporated to handle the uncertainty in experts' judgments. Fuzzy logic serves as an effective decision-making tool that assists in handling uncertainty while preserving available quantitative data (Chen and Gorla, 1998). In this study, the process of determining the priority weights for each dimension and CSF of e-learning using FAHP involved the following steps proposed by Mahad et al. (2019) and Yousif and Shaout (2018)

Step 1. Construction of hierarchy structure for ranking CSFs of e-learning.

Figure 1 shows the hierarchy structure for ranking CSFs of e-learning.

Step 2. Construction of the pairwise comparison matrix for each dimension and factor.

The triangular fuzzy numbers TFNs were used to express the judgment of experts.

Definition 1

A triangular fuzzy number is a specific fuzzy number represented as $\widetilde{M} = (l_i, m_i, u_i)$, $l_i \leq m_i \leq u_i$, where l_i, m_i, u_i are lower, middle and upper values of the fuzzy number $\widetilde{M}$ respectively (Shapiro and Koissi 2017).

Step 3. Checking and assessing the consistency of each expert's responses.

Definition 2

Consistency ratio, ($C.R$) given as: $C.R = \frac{C.I}{R.I}$ is a measure used to assess the reliability of judgments in pairwise comparison matrices, where $C.I$ the consistency index calculated as $C.I = \frac{\lambda_{max} - n}{n-1}$ and RI is the random index given as $R.I = \frac{1.98(n-2)}{n}$, λ_{max} is the priority vector and n is the judgment matrix size. This study accepted $CR \leq 10\%$

Table 1: CSFs of e-learning

Dimension	Code	CSF	Code
Student	D1	Attitude	S1
		Motivation	S2
		Self-efficacy	S3
		Interaction with other students	S4
		Experience in the use of technology	S5
Teacher	D2	Attitude	T1
		ICT skills	T2
		Ease language communication	T3
		Appropriate timely feedback	T4
		Experience in the use of technology	T5
Design and content	D3	Understandable content	DC1
		Availability	DC2
		Flexibility	DC3
		Accessibility	DC4
		Collaborative learning	DC5
System and Technology	D4	Appropriate system	ST1
		Ease of access	ST2
		Valid assessment	ST3
		Interactive learning	ST4
		Technical support for users	ST5
Institutional Management	D5	Infrastructure readiness	IM1
		Financial readiness	IM2
		Training of users	IM3
		Support for department	IM4
		Ethical and legal issues	IM5

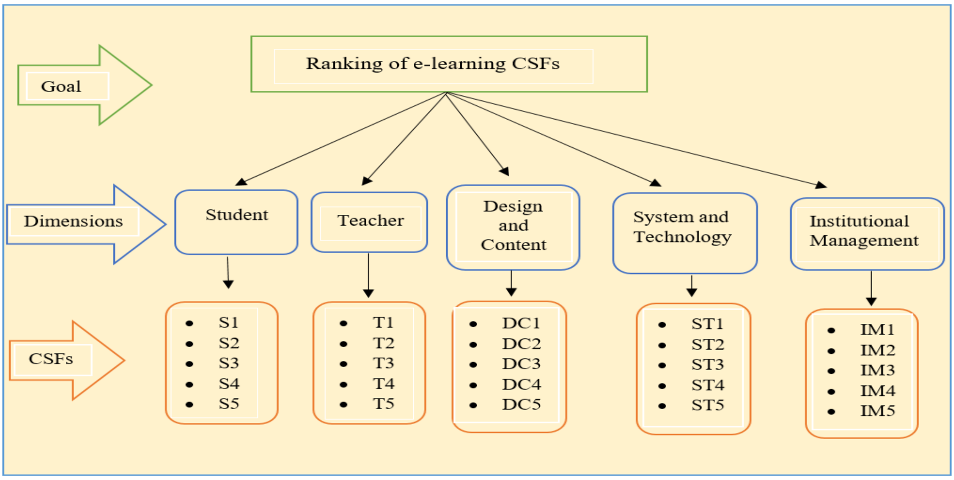

Figure 1: The hierarchical framework for e-learning critical success factors (CSFs)

Step 4. Aggregation of the consistent responses.

After checking the consistency of each individual pairwise comparison response from the DMs and excluding the inconsistent judgments, the step followed was to collectively combine the response values provided by DMs in the relevant matrix into a single value. Aggregation is necessary to achieve a group consensus of the DMs since each matrix assesses one DM. The Geometric Mean (GM) was then applied to the aggregate judgments of all the DMs defined by

$$x_{ij} = \left(\Pi_{k=1}^{K} x_{ijk}\right)^{\frac{1}{K}} = \left(\left(\Pi_{k=1}^{K} l_{ijk}\right)^{\frac{1}{K}}, \left(\Pi_{k=1}^{K} m_{ijk}\right)^{\frac{1}{K}}, \left(\Pi_{k=1}^{K} u_{ijk}\right)^{\frac{1}{K}}\right) \quad (1)$$

where $\left(l_{ijk}, m_{ijk}, u_{ijk}\right)$ is the fuzzy evaluation of sample members k $(k = 1,2, \dots \dots . . K)$. As an example, Table 2 presents the aggregated fuzzy pairwise comparison matrix for the students' dimension, based on experts' evaluations of five associated CSFs.

Table 2: The aggregated fuzzy pairwise comparison matrix for students' dimension

	S1	S2	S3	S4	S5
S1	[1.0, 1.0, 1.0]	[0.9, 1.2, 1.6]	[0.7, 1.0, 1.4]	[0.9, 1.2, 1.5]	[0.7, 1.0, 1.4]
S2	[0.6, 0.8, 1.0]	[1.0, 1.0, 1.0]	[0.7, 1.0, 1.4]	[0.6, 1.1, 1.6]	[0.6, 0.8, 1.2]
S3	[0.7, 1.0, 1.4]	[0.7, 1.0, 1.2]	[1.0, 1.0, 1.0]	[0.7, 1.1, 1.6]	[0.7, 1.1, 1.3]
S4	[0.7, 0.8, 1.1]	[0.6, 1.0, 1.6]	[0.5, 0.8, 1.4]	[1.0, 1.0, 1.0]	[0.5, 0.7, 1.0]
S5	[0.7, 1.0, 1.5]	[0.8, 1.2, 1.6]	[0.7, 1.0, 1.4]	[0.8, 0.9, 1.1]	[1.0, 1.0, 1.0]

Step 5. Calculation of the value of fuzzy synthetic extent using the equation

After aggregating consistent decisions in a combined result, the priorities were estimated using synthetic extent analysis. The extent analysis based on Chang (1996) technique is determined by the Fuzzy synthetic extent value S_i with respect to the i^{th} factor defined as

$$S_i = \sum_{j=1}^{m} M_{gi}^{j} \otimes \left[\sum_{i=1}^{n}\sum_{j=1}^{m} M_{gi}^{j}\right]^{-1} \quad (2)$$

where

$$\sum_{j=1}^{m} M_{gi}^{j} = \left(\sum_{j=1}^{m} l_j, \sum_{j=1}^{m} m_j, \sum_{j=1}^{m} u_j\right)$$

$\sum_{i=1}^{n}\sum_{j=1}^{m} M_{gi}^{j}$ is attained by executing the fuzzy addition operation of $M_{gi}^{j}(j = 1,2, \dots)$ such that

$$\sum_{i=1}^{n}\sum_{j=1}^{m} M_{gi}^{j} = \left(\sum_{i=1}^{n} l_i, \sum_{i=1}^{n} m_i, \sum_{i=1}^{n} u_i\right) \quad (3)$$

And its inverse $\left[\sum_{i=1}^{n}\sum_{j=1}^{m} M_{gi}^{j}\right]^{-1}$ is calculated as

$$\left[\sum_{i=1}^{n}\sum_{j=1}^{m} M_{gi}^{j}\right]^{-1} = \left(\frac{1}{\sum_{i=1}^{n} u_i}, \frac{1}{\sum_{i=1}^{n} m_i}, \frac{1}{\sum_{i=1}^{n} l_i}\right) \quad (4)$$

Step 6. Determination of the degree of possibility

Definition 3

For two triangular fuzzy numbers $M_1 = (l_1, m_1, u_1)$ and $M_2 = (l_2, m_2, u_2)$,the degree of possibility of $M_1 \geq M_2$is defined as$M_1 \geq M_2 = SUP_{y \geq x}\left[min\left(\mu_{M_2}(x), \mu_{M_1}(y)\right)\right]$.

Can be equivalently expressed as $V(M_1 \geq M_2) = hgt(M_1 \cap M_2) = \mu_{M_1}(d)$

$$\mu_{M_1}(d) = \begin{cases} 1 & if\ m_1 \geq m_2 \\ 0 & if\ l_2 \geq u_1 \\ \dfrac{(l_2 - u_1)}{(m_1 - u_1) - (m_2 - l_2)} & other\ wise \end{cases} \quad (5)$$

where d is the ordinate of the highest intersection point between μ_{M_1} andμ_{M_2}.

Step 7. Calculation of weight of each dimension and factor by obtaining the minimum degree of possibility

$d(A_i) = minV(M_i \geq M_k)$

where $k = 1,2, \dots\dots, n \dots\dots\dots$. and $k \neq i$

$W' = \left(d(A_1), d(A_2), \dots\dots, d(A_n)\right)^T$ is the weight vector that represents the relative importance of each dimension and factor.

The final weights are then represented by the normalized weight vector W of all factors in the hierarchy level.

After obtaining FAHP weights, TOPSIS method ranked the e-learning CSFs. A decision matrix was created from importance rating by students, mathematics teachers and IT specialists. From 270 questionnaires distributed, 169 were completed. Rating ranged from 1-5. Mean scores were calculated and matrices normalized. Each normalized weight was then evaluated by multiplying the normalized decision matrix elements with corresponding weights using the equation below.

The TOPSIS model

$$V = V_{ij} = W_j \times R_{ij} \quad (6)$$

By using the equations below the ideal negative and positive solutions are determined.

$$A^{+} = \{V_1^{+}, V_2^{+}, V_3^{+}, \ldots . V_n^{+}\} \quad (7$$
where $V_j^{+} = \{(\max(V_{ij})\ \text{if } j \in J);\ (\min(V_{ij})\ \text{if } j \in J')\}$

$$A^{-} = \{V_1^{-}, V_2^{-}, V_3^{-}, \ldots . V_n^{-}\} \ldots\ldots\ldots\ldots \quad (8)$$
where $V_j^{-} = \{(\min(V_{ij})\ \text{if } j \in J);\ (\max(V_{ij})\ \text{if } j \in J')\}$

The length of each factor from negative ideal solution and positive ideal solution is determined with respect to each dimension by using equations;

$$S^{+} = \sqrt{\sum_{j=1}^{n} \left(V_{ij} - V^{+}\right)^2} \quad \text{for } i = 1 \ldots m \quad (9)$$

$$S^{-} = \sqrt{\sum_{j=1}^{n} \left(V_{ij} - V^{-}\right)^2} \quad \text{for } i = 1 \ldots\ldots . m \quad (10)$$

For each competitive CSF the relative closeness of the potential location with respect to the ideal solution is computed by;

$$P_i = \frac{S_i^{-}}{S_i^{+} + S_i^{-}} \qquad 0 \le P_i \le 1 \quad (11)$$

The ranking was done by using P_i value, the higher value of P_i means the higher the ranking order and alternative can be described as better in terms of performance. Ranking of the preference in descending order thus allows relatively better performances to be compared.

Normalization Methods

In this study, the four commonly used normalization methods used in MCDM problems were applied in ranking the CSFs of e-learning. The methods are vector normalization (N1), linear scale transformation (Max-Min) (N2), linear scale transformation (Max) (N3), and linear scale transformation (Sum) (N4) as they are discussed below. The consistency of the normalization methods was then analyzed by employing four conditions adopted from Çelen (2014).

Condition 1: Normalization methods should produce performance measures with comparable statistical properties, such as means, standard deviations, minimum values, and maximum values.

Condition 2: Different normalization methods should consistently identify the same set of critical success factors (CSFs) as the most and least influential factors.

Condition 3: The ranking of CSFs should be approximately the same across different normalization methods.

Condition 4: The performance scores generated by different normalization methods should be similar for the same CSFs.

Vector Normalization (N1)

Vector normalization is the process of dividing each rating of the decision matrix by its norm.

The normalized value R_{ij} is obtained by

$R_{ij} = \frac{d_{ij}}{\sqrt{\sum_{i=1}^{m} d_{ij}^2}}$, For beneficiary attributes and

$R_{ij} = 1 - \frac{d_{ij}}{\sqrt{\sum_{i=1}^{m} d_{ij}^2}}$, For non-beneficiary attributes

Where d_{ij} is the performance rating of i^{th} factor for attribute C_j.

Linear Scale Transformation (Max-Min) (N2)

This method takes into account both maximum and minimum performance ratings of attributes during calculations. The normalized value R_{ij} is obtained by

$R_{ij} = \frac{d_{ij} - d_j^{min}}{d_j^{max} - d_j^{min}}$, For benefit attributes and

$R_{ij} = \frac{d_j^{max} - d_{ij}}{d_j^{max} - d_j^{min}}$, For non-benefit attributes

Where d_j^{max} is the maximum performance rating among alternatives for attribute C_j and d_j^{min} is the minimum performance rating among alternatives for attribute C_j

Linear Scale Transformation (Max) (N3)

This method divides the performance ratings of each attribute by the maximum performance rate for that attribute. The normalized value R_{ij} is obtained by

$R_{ij} = \frac{d_{ij}}{d_j^{max}}$, for benefit attributes and

$R_{ij} = 1 - \frac{d_{ij}}{d_j^{max}}$, for non-benefit attributes

where d_j^{max} is the maximum performance rating among alternatives for attribute C_j

Linear Scale Transformation (Sum) (N4)

This method divides the performance rating of each attribute by the sum of the performance rating for that attribute. The normalized value R_{ij} is given as

$R_{ij} = \frac{d_{ij}}{\sum_{j=1}^{n} d_j}$, for benefit attributes and

$R_{ij} =$, for non-benefit attributes

where d_j is the maximum performance rating among alternatives for attribute C_j

Results

The purpose of this study was to compare four commonly used normalization procedures in MCDM when ranking the CSFs of e-learning. The study compared relative closeness coefficients obtained from each normalization method. The ranking for each normalization method can be observed in Table 3 below.

Table 3 Ranking of CSFs of e-learning by different normalization models

CSFs	N1	N2	N3	N4
S1	5	6	5	5
S2	23	8	19	19
S3	21	16	21	21
S4	17	7	12	12
S5	1	1	1	1
T1	10	3	7	6
T2	14	13	14	14
T3	18	18	17	17
T4	11	23	20	20
T5	4	14	6	7
DC1	9	2	4	3
DC2	20	4	11	11
DC3	19	15	18	18
DC4	12	19	15	15
DC5	7	11	9	9
ST1	3	5	2	2
ST2	13	17	13	13
ST3	25	22	25	25
ST4	8	9	8	8
ST5	16	10	16	16

IM1	22	24	23	23
IM2	15	25	24	24
IM3	24	20	22	22
IM4	6	21	10	10
IM5	2	12	3	4

The comparison was based on four key conditions to ensure the consistency and reliability of the normalization methods. In condition one, the distribution properties, mean, standard deviation, maximum, and minimum of closeness coefficients for each normalization method were assessed (see Table 4).

Table 4 Statistical distribution properties of closeness coefficient values.

	N1	N2	N3	N4
Mean	0.46	0.51	0.44	0.44
STDV	0.17	0.15	0.20	0.20
Max	0.80	0.81	0.92	0.92
Min	0.20	0.16	0.11	0.11

These distribution properties show insufficient information for similarity. The condition was then tested by applying Kolmogorov–Smirnov (KS) test statistics where D-values and P-values were calculated (see Table 5).

Table 5 Kolmogorov–Smirnov test statistics.

	D-Value	P-Value
N1-N2	0.004	0.969816
N1-N3	0.026	1
N1-N4	0.030	1
N2-N3	0.029	1
N2-N4	0.034	1
N3-N4	0.005	0.99674

The low D-Values and high p-values all≥ 0.9698 show no significant difference between the distributions of closeness coefficients for N1, N2, N3, and N4. Therefore, condition one was satisfied.

In the second condition, the most and the least influential factors were identified for each normalization method. The results are presented in Table 6 below.

Table 6: The most and the least influential factors

	The most influential factor	The least influential factor
N1	S5	ST3
N2	S5	IM2
N3	S5	ST3
N4	S5	ST3

All normalization methods identified students' experience in the use of technology (S5) as the most influential factor. According to N2, financial readiness (IM2) was identified as the least influential factor, while the rest of the models identified valid assessment (ST3). S5's consistent identification as the most influential CSF validates its importance for e-learning success. The divergence in identifying the least influential factor through N2 emphasizes the need for careful normalization method selection.

To satisfy the third condition, the ranking consistency was checked by using Spearman's rank correlation coefficients. Table 7 below shows the results.

Table 7 Correlations between rankings of different normalization models.

	N1	N2	N3	N4
N1	1.00	0.41	0.86	0.85
N2		1.00	0.76	0.78
N3			1.00	0.99
N4				1.00

The results indicate that all models produced similar rankings except for N1-N2, which exhibited a moderate correlation of 0.41. This implies that N2 is not a reliable method compared to the other three, particularly with the FAHP-TOPSIS model. Also, this shows the importance of selecting a normalization method that maintains ranking stability in decision-making.

To meet the fourth condition, the correlation between the closeness coefficients was evaluated by using Pearson correlation coefficients. The results are as seen in Table 8.

Table 8 Correlations between closeness coefficient values of different normalization models.

	N1	N2	N3	N4
N1	1.00	0.70	0.93	0.92
N2		1.00	0.76	0.76
N3			1.00	0.94
N4				1.00

The lowest correlation is found between N1 and N2, as well as between N2 and both N3 and N4, highlighting slight deviation of N2 from the others. The high Pearson correlation coefficient among the relative closeness coefficients confirms that the same CSFs receives similar performance scores across all normalization methods, implying the satisfaction of the condition.

Discussion

From the comparative analysis-based results obtained, the study emphasizes the significance of choosing an appropriate normalization method in Multi-Criteria Decision Making (MCDM) models, especially in the FAHP-TOPSIS framework. The consistent results of Vector (N1), Max (N3) and Sum (N4) normalization methods across all four conditions demonstrate their reliability in ranking CSFs for e-learning, with the student's experience in using technology being the most influential factor while valid assessment is the least influential. On the other hand, the inconsistent behavior of Max-Min (N2), particularly in identifying the least influential factor and its moderate correlation with N1, raises concerns about its suitability in certain situations.

The consistency of N1, N3, and N4 in ranking CSFs across conditions indicates robustness in handling decision matrix variations by maintaining proportional relationships among factors. The N2's inconsistency with moderate correlation (0.41) to N1 suggests sensitivity to extreme values. The least relevant rankings of N2, contrary to research identifying N4 as the weakest, highlight the context-dependent nature of normalization in MCDM applications. In Tanzania's secondary schools, where CSFs of e-learning show uneven distributions, the sensitivity of N2 to range likely distorts relationships among factors, aligning with Çelen (2014)'s caution against N2 in irregular datasets. The contrast with studies like Chakraborty and Yeh (2009) and Vafaei et al. (2016), which found N4 least reliable, derives from domain differences; financial data require precision, whereas educational decisions prioritize interpretability. This implies that normalization is a strategic choice that must align the data and decision context.

Conclusion

This study evaluated the impact of different normalization methods on decision outcomes using MCDM approaches. The study developed the FAHP-TOPSIS model to assess and rank the Critical success factors (CSFs) of e-learning. Eleven decision makers conducted pairwise comparisons for twenty-five CSFs across five different dimensions using the FAHP model. Triangular fuzzy numbers from these comparisons were used to calculate the weights of dimensions with their corresponding CSFs. In the TOPSIS stage, a decision matrix was created, followed by the generation of four TOPSIS models using four normalization methods: vector normalization (N1), Max-Min (N2), Max (N3), and Sum (N4). Positive and negative ideal solutions were defined, and the distance of each CSF from these solutions was calculated. The CSFs were ranked based on the closeness coefficients derived from these distances in each TOPSIS model. The ranking results were compared using the consistency conditions adopted.

According to the FAHP-TOPSIS model results, students' experience in technology and legal and ethical issues were identified as the most influential factors, while valid assessment was the least influential. Students' experience in technology was also ranked first in all normalization models.

These findings highlight where e-learning investments in secondary schools should be prioritized: students must first be digitally prepared, and their online experiences must be safeguarded through policies that protect their rights and ensure safety; otherwise, other e-learning components may remain underutilized.

The ranking results of N1, N3, and N4 were consistent, as they satisfied all consistency conditions. Conversely, N2 identified a distinct least influential factor and showed a moderate correlation with N1. This may indicate that N2 is sensitive to certain data characteristics. This concludes that normalization procedures may affect the decision outcome of an MCDM method. The findings highlight the importance of selecting suitable normalization methods to ensure robust decision-making processes in MCDM. The findings also justified the application of the vector normalization procedure with the TOPSIS model. Moreover, the results align with Vafaei et al. (2016), who also observed that the choice of normalization methods can significantly impact MCDM outcomes. They also support Başaran and Haruna's (2017) findings that robust normalization enhances the reliability of e-learning evaluations. Future research should focus on conducting sensitivity analysis to evaluate the impact of normalization methods on MCDM methods.

Declaration of interest

The authors declare no conflicts of interest.

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