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Comparative study on the application of GRA, PSI and hybrid PSI-GRA techniques for optimising turning parameters in multi-criteria problems

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Keywords

Multi-criteria decision-making; Grey relational analysis, Preference selection index; Turning process; Optimization

Abstract

The optimum turning parameters help to justify the investment in machine tools for manufacturing industries. However, solving optimisation models to achieve optimum turning parameters in multi-criteria environment is complex leading to the application of multi-criteria decision-making techniques. This paper compares three multi-criteria decision-making techniques in determining optimum turning process parameters namely spindle speed, feed rate and depth of cut. The performance criteria are material removal rate, consumed electrical energy and tool wear rate. Example data for turning process were obtained from other studies while the analyses for multi-criteria decision-making were performed using grey relational analysis (GRA), preference selection index (PSI) and hybrid PSI-GRA techniques. The results show that, hybrid PSI-GRA is the most effective technique taking advantage of the strengths of both GRA and PSI techniques. The optimum process parameters are spindle speed N = 355 rpm, feed rate f = 0.05 mm/rev and depth of cut d = 1.5 mm with associated material removal rate MRR = 5846.11 mm^3/min; consumed electrical energy CEE = 0.0871 kWh; and tool wear rate TWR = 0.0415 μm/min. These parameters are considered efficient and can help to achieve higher performance of turning operations including lower cost, higher productivity and higher product quality.

Introduction

Turning operation is one of the primary metal machining processes which involve the removal of material by means of relative motions between the cutting tool and the raw material. In order to achieve efficient turning operation, a combination of machining parameters such as spindle speed, feed rate and depth of cut is essentially established. However, it is difficult to achieve optimal machining parameters in multiple criteria environments. That is why most of machining studies tend to deal more with single-criterion problems than multi-criteria problems. This is due to the complexity of multi-criteria problems which try to achieve optimum machining parameters under several conflicting performance criteria. Although multi-criteria optimisation models for machining operations are complex and need robust procedures to solve, the application of other numerical multicriteria decision making techniques has emerged. Some of these techniques reported by Triantaphyllou and Sanchez (1997), Maniya and Bhatt (2010), Ishizaka and Siraj (2018), and Yazdani et al. (2019) include Weighted Sum Model (WSM), Weighted Product Model (WPM),

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Weighted Aggregated Sum Product Assessment (WASPAS), Evaluation Based on Distance Average Solution (EDAS), Combinative Distance-Based Assessment (CODAS), Complex Proportional Assessment (COPRAS), Multi-Objective Optimisation Ratio Analysis (MOORA), Grey Relational Analysis (GRA), Elimination and Choice Expressing Reality (ELECTRE), Preference Ranking Organisation Method for Enrichment Evaluation (PROMETHEE), Technique for Order Preference by Similarity to Ideal Solution (TOPSIS), "Visekriterijumska optimizacija I Kompromisno Resenje" in Serbian (VIKOR), Simple Multi-Attribute Rating Technique (SMART), Measuring Attractiveness by a Categorical Based Evaluation Technique (MACBETH), Analytic Hierarchy Process (AHP), and Preference Selection Index (PSI). Maniya and Bhatt (2010) implemented PSI method for material selection to meet design requirements. Kumar et al. (2017) solved a multi-criteria optimisation problem of energy consumption, surface roughness and material removal rate of turning operations. Taguchi orthogonal array method was used to design a series of experiments followed by analysis with TOPSIS method using AHP weights, equal weights and Entropy weights to determine optimal turning parameters.

The work by Umamaheswarrao et al. (2018) implemented machining experiments designed using Taguchi method and deployed a hybrid GRA method coupled with principal component analysis (PCA) to solve a multi-objective optimisation problem for turning operations in order to provide optimum set of cutting parameters to meet surface quality, machining force and surface temperature requirements. Related studies have been reported more recently. Rathod et al. (2023) carried out experimental study and used the Taguchi and TOPSIS methods for finding optimum process parameters in turning operations to maximise tool life and minimise production time and surface roughness. Mahajan and Chakraborty (2024) applied PSI method to determine the best alternative of three different work-tool combinations for machining operation. Saeed et al. (2024) conducted a comparative analysis of two multi-criteria decision analysis methods, GRA and TOPSIS to determine sustainable machining parameters. Yallese et al. (2025) performed multi-objective optimisation using three multi-criteria decision-making techniques namely TOPSIS, Combined Compromise Solution (COCOSO), and Evaluation by Area-based Method of Ranking (EAMR). Atif et al. (2025) developed a framework that combine multi-objective optimisation (MOO) using NSGA-II, multi-criteria decision-making using GRA, and experimental design techniques for improving machining operations. Ryang et al. (2025) developed a multiple performances optimisation methodology for computer numerical controlled (CNC) machining based on Taguchi method, multi-criteria decision-making (MCDM) and multiple regression (MR) model. Several studies have found GRA to be useful while PSI remains underexplored. However, the GRA method employs equal criteria weights which may be subjective. This study therefore analyses the application of GRA, PSI and the hybrid PSI-GRA for finding the optimum process parameters in turning operations. It highlights a multi-criteria decision-making problem with cutting electrical energy, material removal rate and tool wear rate as performance criteria while spindle speed, feed rate and depth of cut are turning process parameters to be optimised.

Methods and Materials

The study implemented GRA, PSI and hybrid PSI-GRA techniques. Firstly, the GRA technique was used to determine the best combination of machining parameters for cutting electrical energy, material removal rate and tool wear rate criteria. GRA technique uses equal weights which may be subjective. Secondly, the PSI technique was used for selecting the best combination of machining parameters that satisfy requirements of all performance criteria. Finally, the weights obtained in the PSI technique was used as inputs to the GRA technique to form a hybrid of PSI and GRA.

GRA employs fours steps to be followed in order to rank the combinations of machining parameters. The steps are as follows (Kuo et al, 2008):

Step 1: Normalise decision matrix to make the data in the range between 0 and 1 using the following equations:

Benefit criterion $$x_i^{*}(k)=\frac{x_i^o(k)-\min x_i^o(k)}{\max x_i^o(k)-\min x_i^o(k)} \quad (1)$$

Non-benefit criterion $$x_i^{*}(k)=\frac{\max x_i^o(k)-x_i^o(k)}{\max x_i^o(k)-\min x_i^o(k)} \quad (2)$$

where $x_i^o(k)$ is the original data, $\max x_i^o(k)$ the maximum value of $x_i^o(k)$, and $\min x_i^o(k)$ imply the minimum value of $x_i^o(k)$.

Step 2: Determine the grey relational coefficient which is calculated as follows.

$$\zeta_i(k)=\frac{\Delta_{min}+\zeta\,\Delta_{max}}{\Delta_{oi}(k)+\zeta\,\Delta_{max}} \quad (3)$$

where $\Delta_{oi(k)}$ is the deviation in sequence from the reference sequence, which is given by:

$$\Delta_{oi}(k)=\left\|x_o^{*}(k)-x_i^{*}(k)\right\| \quad (4)$$

$$\Delta_{max}=\max_{j\in i}\max_{\forall k}\left\|x_o^{*}(k)-x_j^{*}(k)\right\| \quad (5)$$

$$\Delta_{min}=\min_{j\in i}\min_{\forall k}\left\|x_o^{*}(k)-x_j^{*}(k)\right\| \quad (6)$$

A value ζ is the distinguishing coefficient, $\zeta \in [0, 1]$ used to determine how close $x_o^*(k)$ is to $x_i^*(k)$. The larger the coefficient, the closer between the two.

Step 3: Determine weighted grey relational grade using the following equation:

$$\gamma_i=\frac{1}{n}\sum_{k=1}^{n} w_k(k)\zeta_i(k) \quad (7)$$

Where $w_k(k)$ is the criteria weights obtained through the vector entropy method. The sum of criteria weights must be 1.

Step 4: Rank the results to determine the best choice with lowest weighted grey relational grade ranked the first.

PSI employs seven steps that should be followed to ensure that the ranking of combinations of machining parameters is made. The steps are as follows (Maniya and Bhatt, 2010):

Step 1: Normalise decision matrix ***R*** to make the data in the range between 0 and 1 using the following standardizing equation:

Benefit criterion $$N_{ij}=\frac{X_{ij}}{X_j^{max}} \quad (8)$$

Non-benefit criterion $$N_{ij}=\frac{X_j^{min}}{X_{ij}} \quad (9)$$

Where X_{ij} is the attribute measure (i = 1, 2, …, N and j = 1, 2, …, M).

Step 2: Compute the mean value of the normalised data by the following equation:

$$N=\frac{1}{n}\sum_{i=1}^{n} N_{ij} \quad (10)$$

Step 3: Compute the preference variation value between every attribute, which is calculated by:

$$\varnothing_j = \sum_{i-1}^{n} \left[N_{ij} - N \right]^2 \tag{11}$$

Step 4: Determine the deviation in preference value by using the following expression:

$$\gamma_j = \left[1 - \varnothing_j \right] \tag{12}$$

Step 5: Compute the overall preference value for every attribute as follows

$$\omega_j = \frac{\gamma_j}{\sum_{j=1}^{m} \gamma_j} \tag{13}$$

Step 6: The preference selection index is calculated for each alternative using the following equation:

$$\theta_i = \sum_{j=1}^{M} X_{ij} \times \omega_j \tag{14}$$

Step 7: Rank the results to determine the best choice with highest preference selection index ranked first.

The hybrid PSI-GRA employs step 1 through step 5 of PSI and then linked by step 3 and step 4 of GRA. As reported earlier, the GRA method employs equal criteria weights which may be subjective. In order to avoid subjectivity in criteria weights, the hybrid PSI-GRA is used whereby the criteria weights obtained in PSI method are used to link with GRA method. Combinations of input process parameters of spindle speed N (rpm), feed rate f (mm/rev) and depth of cut d (mm) associated with output criteria of material removal volume Q (mm^3), consumed power P (kW), machining time t (min) and tool wear w (µm) were used to determine the optimum parameters. The example data for these combinations were borrowed from Shishira (2022) and they are presented in Table 1.

Table 1: Turning parameters for material removal volume, consumed power, machining time and tool wear

S/N	Input variables			Output variables			
	N (rpm)	f (mm/rev)	d (mm)	Q (mm^3)	P (kW)	t (min)	w (µm)
1	600	0.2	1.5	20622.93	1.66	2.0	0.251
2	600	0.2	0.8	9713.38	1.30	2.0	0.142
3	600	0.2	0.4	5504.46	1.22	2.0	0.081
4	600	0.1	1.5	20791.08	1.30	4.0	0.208
5	600	0.1	0.8	11406.37	1.20	4.0	0.109
6	600	0.1	0.4	5261.15	1.06	4.0	0.058
7	600	0.05	1.5	23517.20	1.06	8.0	0.201
8	600	0.05	0.8	12270.06	1.02	8.0	0.104
9	600	0.05	0.4	6343.95	0.96	8.1	0.069
10	355	0.2	1.5	21825.48	1.40	3.7	0.155
11	355	0.2	0.8	11462.42	1.38	3.4	0.099
12	355	0.2	0.4	5839.49	1.28	3.4	0.051
13	355	0.1	1.5	21662.42	1.40	6.7	0.114
14	355	0.1	0.8	12347.77	1.28	6.7	0.071
15	355	0.1	0.4	5750.32	1.18	6.8	0.036
16	355	0.05	1.5	22601.27	1.20	13.7	0.104
17	355	0.05	0.8	12332.48	1.16	13.5	0.066

18	355	0.05	0.4	5543.95	1.14	13.4	0.033
19	150	0.2	1.5	17157.96	0.90	7.9	0.099
20	150	0.2	0.8	8215.29	0.80	7.9	0.064
21	150	0.2	0.4	3287.90	0.78	7.9	0.030
22	150	0.1	1.5	17777.07	0.80	16.0	0.058
23	150	0.1	0.8	6387.26	0.76	15.7	0.033
24	150	0.1	0.4	2439.49	0.78	15.8	0.018
25	150	0.05	1.5	18699.36	0.78	32.5	0.041
26	150	0.05	0.8	11978.34	0.76	32.1	0.028
27	150	0.05	0.4	7214.01	0.78	32.3	0.015

This study deals with the combinations of process parameters connected with performance criteria of material removal rate, consumed electrical energy and tool wear rate for the turning operations. In this case, the material removal volume, consumed power, machining time and tool wear in Table 1 were used to calculate material removal rate (MRR), consumed electrical energy (CEE) and tool wear rate (TWR). Equations (15), (16) and (17) represent calculations of material removal rate, consumed electrical energy and tool wear rate respectively.

$$MRR = \frac{Q}{t} \tag{15}$$

$$CEE = P \times t \tag{16}$$

$$TWR = \frac{w}{t} \tag{17}$$

Where Q is the material removal volume (mm^3), P = consumed power (kW), t = machining time (min) and w = tool wear (μm). Therefore, the combinations of turning parameters associated with material removal rate MRR (mm^3/min), consumed electrical energy CEE (kWh) and tool wear rate TWR (μm/min) are presented in Table 2.

Table 2: Turning parameters for material removal rate, consumed electrical energy and tool wear rate

S/N	Input variables *N* (rpm)	*f* (mm/rev)	*d* (mm)	Output variables *MRR* (mm^3/min)	*CEE* (kWh)	*TWR* (μm/min)
1	600	0.2	1.5	10311.47	0.0553	0.1255
2	600	0.2	0.8	4856.69	0.0433	0.0710
3	600	0.2	0.4	2752.23	0.0407	0.0405
4	600	0.1	1.5	5197.77	0.0867	0.0520
5	600	0.1	0.8	2851.59	0.0800	0.0273
6	600	0.1	0.4	1315.29	0.0707	0.0145
7	600	0.05	1.5	2951.95	0.1407	0.0252
8	600	0.05	0.8	1527.39	0.1366	0.0129
9	600	0.05	0.4	788.07	0.1288	0.0086
10	355	0.2	1.5	5846.11	0.0871	0.0415
11	355	0.2	0.8	3421.62	0.0771	0.0296
12	355	0.2	0.4	1734.50	0.0718	0.0151
13	355	0.1	1.5	3249.36	0.1556	0.0171
14	355	0.1	0.8	1852.17	0.1422	0.0107
15	355	0.1	0.4	851.90	0.1328	0.0053
16	355	0.05	1.5	1645.72	0.2747	0.0076
17	355	0.05	0.8	915.78	0.2604	0.0049
18	355	0.05	0.4	414.76	0.2540	0.0025
19	150	0.2	1.5	2185.73	0.1178	0.0126

20	150	0.2	0.8	1046.53	0.1047	0.0082
21	150	0.2	0.4	416.19	0.1027	0.0038
22	150	0.1	1.5	1112.23	0.2131	0.0036
23	150	0.1	0.8	407.26	0.1987	0.0021
24	150	0.1	0.4	154.89	0.2048	0.0011
25	150	0.05	1.5	576.25	0.4219	0.0013
26	150	0.05	0.8	373.16	0.4066	0.0009
27	150	0.05	0.4	223.11	0.4203	0.0005

Results and Discussion

The optimisation of turning process parameters involves the application of GRA, PSI and PSI-GRA techniques. The normalised GRA decision matrix is presented in Table 3. The normalisation was followed by determining the grey relational coefficient, grey relational grade and the best combination as shown in Table 4. The GRA results in Table 4 show that, combination 10 is highly ranked; thus, the optimum turning process parameters are speed N = 355 rpm, feed rate f = 0.05 mm/rev and depth of cut d = 1.5 mm with associated material removal rate MRR = 5846.11 mm^3/min; consumed electrical energy CEE = 0.0871 kWh; and tool wear rate TWR = 0.0415 μm/min. The normalised PSI decision matrix is shown in Table 5. The normalisation was followed by determining preference variation value, deviation in preference value, preference selection index and the best combination as shown in Table 6. The PSI results in Table 6 show that, combination 1 is highly ranked; thus, the optimum turning parameters are spindle speed N = 600 rpm, feed rate f = 0.2 mm/rev and depth of cut d = 1.5 mm with associated material removal rate MRR = 10311.47 mm^3/min; consumed electrical energy CEE = 0.0553 kWh; and tool wear rate TWR = 0.1255 μm/min. The hybrid PSI-GRA employed the normal GRA method but used the weight criteria obtained in the PSI method. The PSI-GRA rankings are presented in Table 7 in comparison with GRA and PSI rankings. The overall results in Table 7 shows that, the best combination with hybrid PSI-GRA method is similar to GRA method i.e., spindle speed N = 355 rpm, feed rate f = 0.05 mm/rev and depth of cut d = 1.5 mm with associated material removal rate MRR = 5846.11 mm^3/min; consumed electrical energy CEE = 0.0871 kWh; and tool wear rate TWR = 0.0415 μm/min. Although the best combination of PSI-GRA is similar to the best combination of GRA and different from the best combination of PSI method, these three techniques have shown ranking divergence in some other combinations of turning parameters. This indicates that different multi-criteria decision-making techniques may provide different solutions. The results also show that PSI selected higher MRR but worse TWR. This may be due to the fact that higher rate of material removed can cause higher friction between the cutting tool and machined surface of the workpiece, hence increased tool wear. Additionally, higher machining parameters tend to increase both MRR and TWR leading to shorter tool life. It is further observed that lower machining parameters tend to increase CEE due to extended machining time causing more energy to be consumed by the machine tool. Owing to the importance of realistic combination of machining parameters for manufacturing shop floor, the application of multi-criteria decision-making techniques becomes necessary.

Table 3: GRA normalisation of criteria weights

S/N	*Process parameters*			*Performance criteria*			*Normalisation*		
	N (rpm)	*f* (mm/rev)	*d* (mm)	*MRR* (mm^3/min)	*CEE* (kWh)	*TWR* *(mm/min)*	*MRR* (mm^3/min)	*CEE* (kWh)	*TWR* (mm/min)
1	600	0.2	1.5	10311.47	0.0553	0.1255	1.0000	0.9615	0.0000
2	600	0.2	0.8	4856.69	0.0433	0.0710	0.4629	0.9930	0.4359
3	600	0.2	0.4	2752.23	0.0407	0.0405	0.2557	1.0000	0.6798
4	600	0.1	1.5	5197.77	0.0867	0.0520	0.4965	0.8793	0.5878
5	600	0.1	0.8	2851.59	0.0800	0.0273	0.2655	0.8968	0.7858
6	600	0.1	0.4	1315.29	0.0707	0.0145	0.1143	0.9213	0.8877
7	600	0.05	1.5	2951.95	0.1407	0.0252	0.2754	0.7375	0.8019
8	600	0.05	0.8	1527.39	0.1366	0.0129	0.1351	0.7484	0.9002
9	600	0.05	0.4	788.07	0.1288	0.0086	0.0623	0.7688	0.9352
10	355	0.2	1.5	5846.11	0.0871	0.0415	0.5603	0.8782	0.6717
11	355	0.2	0.8	3421.62	0.0771	0.0296	0.3216	0.9046	0.7674
12	355	0.2	0.4	1734.50	0.0718	0.0151	0.1555	0.9183	0.8826
13	355	0.1	1.5	3249.36	0.1556	0.0171	0.3047	0.6986	0.8669
14	355	0.1	0.8	1852.17	0.1422	0.0107	0.1671	0.7336	0.9185
15	355	0.1	0.4	851.90	0.1328	0.0053	0.0686	0.7584	0.9611
16	355	0.05	1.5	1645.72	0.2747	0.0076	0.1468	0.3861	0.9431
17	355	0.05	0.8	915.78	0.2604	0.0049	0.0749	0.4237	0.9645
18	355	0.05	0.4	414.76	0.2540	0.0025	0.0256	0.4404	0.9840
19	150	0.2	1.5	2185.73	0.1178	0.0126	0.2000	0.7978	0.9028
20	150	0.2	0.8	1046.53	0.1047	0.0082	0.0878	0.8321	0.9385
21	150	0.2	0.4	416.19	0.1027	0.0038	0.0257	0.8373	0.9733
22	150	0.1	1.5	1112.23	0.2131	0.0036	0.0943	0.5476	0.9747
23	150	0.1	0.8	407.26	0.1987	0.0021	0.0248	0.5855	0.9869
24	150	0.1	0.4	154.89	0.2048	0.0011	0.0000	0.5695	0.9946
25	150	0.05	1.5	576.25	0.4219	0.0013	0.0415	0.0000	0.9936
26	150	0.05	0.8	373.16	0.4066	0.0009	0.0215	0.0400	0.9967
27	150	0.05	0.4	223.11	0.4203	0.0005	0.0067	0.0040	1.0000

Table 4: Weighted grey relational grade (GRG) and ranking for best combination

S/N	*Process parameters*			*Reference sequence*			*Deviation in sequence*			*Grey relational coefficient*			*GRG*	*Rank*
	N (rpm)	*f* (mm/rev)	*d* (mm)	*MRR* (mm^3/min)	*CEE* (*kWh*)	*TWR* mm/min	*MRR* (*mm*^{3}/min)	*CEE* (kWh)	*TWR* mm/min	*MRR* (mm^3/min)	*CEE* (kWh)	*TWR* mm/min		
1	600	0.2	1.5	0.0000	0.0000	0.0000	1.0000	0.9615	0.0000	0.3333	0.3421	1.0000	0.5579	19
2	600	0.2	0.8	0.0000	0.0000	0.0000	0.4629	0.9930	0.4359	0.5192	0.3349	0.5343	0.4623	4
3	600	0.2	0.4	0.0000	0.0000	0.0000	0.2557	1.0000	0.6798	0.6616	0.3333	0.4238	0.4724	7
4	600	0.1	1.5	0.0000	0.0000	0.0000	0.4965	0.8793	0.5878	0.5017	0.3625	0.4596	0.4409	2
5	600	0.1	0.8	0.0000	0.0000	0.0000	0.2655	0.8968	0.7858	0.6532	0.3580	0.3889	0.4662	5
6	600	0.1	0.4	0.0000	0.0000	0.0000	0.1143	0.9213	0.8877	0.8140	0.3518	0.3603	0.5082	12
7	600	0.05	1.5	0.0000	0.0000	0.0000	0.2754	0.7375	0.8019	0.6448	0.4041	0.3840	0.4772	8
8	600	0.05	0.8	0.0000	0.0000	0.0000	0.1351	0.7484	0.9002	0.7872	0.4005	0.3571	0.5144	13
9	600	0.05	0.4	0.0000	0.0000	0.0000	0.0623	0.7688	0.9352	0.8891	0.3941	0.3484	0.5433	16
10	355	0.2	1.5	0.0000	0.0000	0.0000	0.5603	0.8782	0.6717	0.4715	0.3628	0.4267	0.4199	1
11	355	0.2	0.8	0.0000	0.0000	0.0000	0.3216	0.9046	0.7674	0.6085	0.3560	0.3945	0.4526	3
12	355	0.2	0.4	0.0000	0.0000	0.0000	0.1555	0.9183	0.8826	0.7627	0.3525	0.3616	0.4918	10
13	355	0.1	1.5	0.0000	0.0000	0.0000	0.3047	0.6986	0.8669	0.6214	0.4172	0.3658	0.4676	6
14	355	0.1	0.8	0.0000	0.0000	0.0000	0.1671	0.7336	0.9185	0.7495	0.4053	0.3525	0.5019	11
15	355	0.1	0.4	0.0000	0.0000	0.0000	0.0686	0.7584	0.9611	0.8793	0.3973	0.3422	0.5391	15
16	355	0.05	1.5	0.0000	0.0000	0.0000	0.1468	0.3861	0.9431	0.7731	0.5643	0.3465	0.5607	20
17	355	0.05	0.8	0.0000	0.0000	0.0000	0.0749	0.4237	0.9645	0.8697	0.5413	0.3414	0.5836	22
18	355	0.05	0.4	0.0000	0.0000	0.0000	0.0256	0.4404	0.9840	0.9513	0.5317	0.3369	0.6060	24
19	150	0.2	1.5	0.0000	0.0000	0.0000	0.2000	0.7978	0.9028	0.7143	0.3853	0.3564	0.4849	9
20	150	0.2	0.8	0.0000	0.0000	0.0000	0.0878	0.8321	0.9385	0.8506	0.3753	0.3476	0.5240	14
21	150	0.2	0.4	0.0000	0.0000	0.0000	0.0257	0.8373	0.9733	0.9511	0.3739	0.3394	0.5542	18
22	150	0.1	1.5	0.0000	0.0000	0.0000	0.0943	0.5476	0.9747	0.8414	0.4773	0.3391	0.5520	17
23	150	0.1	0.8	0.0000	0.0000	0.0000	0.0248	0.5855	0.9869	0.9527	0.4606	0.3363	0.5826	21
24	150	0.1	0.4	0.0000	0.0000	0.0000	0.0000	0.5695	0.9946	1.0000	0.4675	0.3345	0.6001	23
25	150	0.05	1.5	0.0000	0.0000	0.0000	0.0415	0.0000	0.9936	0.9234	1.0000	0.3348	0.7520	26
26	150	0.05	0.8	0.0000	0.0000	0.0000	0.0215	0.0400	0.9967	0.9588	0.9259	0.3341	0.7388	25
27	150	0.05	0.4	0.0000	0.0000	0.0000	0.0067	0.0040	1.0000	0.9867	0.9921	0.3333	0.7700	27

Table 5: PSI normalisation of criteria weights

S/N	*Process parameters*			*Performance criteria*			*Normalisation*		
	N (rpm)	*f* (mm/rev)	*d* (mm)	*MRR* (mm^3/min)	*CEE* (kWh)	*TWR* (mm/min)	*MRR* (mm^3/min)	*CEE* (kWh)	*TWR* (mm/min)
1	600	0.2	1.5	10311.47	0.0553	0.1255	1.000	0.735	0.004
2	600	0.2	0.8	4856.69	0.0433	0.0710	0.471	0.938	0.007
3	600	0.2	0.4	2752.23	0.0407	0.0405	0.267	1.000	0.011
4	600	0.1	1.5	5197.77	0.0867	0.0520	0.504	0.469	0.009
5	600	0.1	0.8	2851.59	0.0800	0.0273	0.277	0.508	0.017
6	600	0.1	0.4	1315.29	0.0707	0.0145	0.128	0.575	0.032
7	600	0.05	1.5	2951.95	0.1407	0.0252	0.286	0.289	0.018
8	600	0.05	0.8	1527.39	0.1366	0.0129	0.148	0.298	0.036
9	600	0.05	0.4	788.07	0.1288	0.0086	0.076	0.316	0.054
10	355	0.2	1.5	5846.11	0.0871	0.0415	0.567	0.467	0.011
11	355	0.2	0.8	3421.62	0.0771	0.0296	0.332	0.528	0.016
12	355	0.2	0.4	1734.50	0.0718	0.0151	0.168	0.566	0.031
13	355	0.1	1.5	3249.36	0.1556	0.0171	0.315	0.261	0.027
14	355	0.1	0.8	1852.17	0.1422	0.0107	0.180	0.286	0.044
15	355	0.1	0.4	851.90	0.1328	0.0053	0.083	0.306	0.087
16	355	0.05	1.5	1645.72	0.2747	0.0076	0.160	0.148	0.061
17	355	0.05	0.8	915.78	0.2604	0.0049	0.089	0.156	0.095
18	355	0.05	0.4	414.76	0.2540	0.0025	0.040	0.160	0.188
19	150	0.2	1.5	2185.73	0.1178	0.0126	0.212	0.345	0.037
20	150	0.2	0.8	1046.53	0.1047	0.0082	0.101	0.389	0.057
21	150	0.2	0.4	416.19	0.1027	0.0038	0.040	0.396	0.122
22	150	0.1	1.5	1112.23	0.2131	0.0036	0.108	0.191	0.128
23	150	0.1	0.8	407.26	0.1987	0.0021	0.039	0.205	0.220
24	150	0.1	0.4	154.89	0.2048	0.0011	0.015	0.199	0.406
25	150	0.05	1.5	576.25	0.4219	0.0013	0.056	0.096	0.367
26	150	0.05	0.8	373.16	0.4066	0.0009	0.036	0.100	0.532
27	150	0.05	0.4	223.11	0.4203	0.0005	0.022	0.097	1.000

Table 6: Preference Selection Index (PSI) and ranking for best combination

S/N	*Process parameters*			*Performance criteria*			*Preference variation value*			*Deviation in preference value*			PSI	*Ranking*
	N (rpm)	*f* (mm/rev)	*d* (mm)	*MRR* (mm^3/min)	*CEE* (kWh)	*TWR* (mm/min)	*MRR* (mm^3/min)	*CEE* (kWh)	*TWR* (mm/min)	*MRR* (mm^3/min)	*CEE* (kWh)	*TWR* (mm/min)		
1	600	0.2	1.5	10311.47	0.0553	0.1255	0.6212	0.1322	0.0170	0.2510	0.3645	0.0009	0.616	1
2	600	0.2	0.8	4856.69	0.0433	0.0710	0.0672	0.3217	0.0162	0.1182	0.4654	0.0017	0.585	2
3	600	0.2	0.4	2752.23	0.0407	0.0405	0.0030	0.3953	0.0150	0.0670	0.4959	0.0029	0.566	3
4	600	0.1	1.5	5197.77	0.0867	0.0520	0.0854	0.0096	0.0156	0.1265	0.2327	0.0023	0.362	5
5	600	0.1	0.8	2851.59	0.0800	0.0273	0.0042	0.0188	0.0137	0.0694	0.2521	0.0043	0.326	8
6	600	0.1	0.4	1315.29	0.0707	0.0145	0.0071	0.0417	0.0104	0.0320	0.2854	0.0081	0.326	9
7	600	0.05	1.5	2951.95	0.1407	0.0252	0.0055	0.0068	0.0133	0.0719	0.1433	0.0047	0.220	14
8	600	0.05	0.8	1527.39	0.1366	0.0129	0.0041	0.0054	0.0096	0.0372	0.1477	0.0091	0.194	19
9	600	0.05	0.4	788.07	0.1288	0.0086	0.0183	0.0031	0.0064	0.0192	0.1566	0.0137	0.189	21
10	355	0.2	1.5	5846.11	0.0871	0.0415	0.1261	0.0091	0.0151	0.1423	0.2315	0.0028	0.377	4
11	355	0.2	0.8	3421.62	0.0771	0.0296	0.0144	0.0245	0.0140	0.0833	0.2618	0.0040	0.349	6
12	355	0.2	0.4	1734.50	0.0718	0.0151	0.0019	0.0380	0.0107	0.0422	0.2808	0.0077	0.331	7
13	355	0.1	1.5	3249.36	0.1556	0.0171	0.0107	0.0121	0.0114	0.0791	0.1297	0.0069	0.216	15
14	355	0.1	0.8	1852.17	0.1422	0.0107	0.0010	0.0073	0.0082	0.0451	0.1418	0.0110	0.198	17
15	355	0.1	0.4	851.90	0.1328	0.0053	0.0167	0.0042	0.0022	0.0207	0.1519	0.0220	0.195	18
16	355	0.05	1.5	1645.72	0.2747	0.0076	0.0027	0.0498	0.0053	0.0401	0.0734	0.0155	0.129	26
17	355	0.05	0.8	915.78	0.2604	0.0049	0.0151	0.0463	0.0015	0.0223	0.0775	0.0240	0.124	27
18	355	0.05	0.4	414.76	0.2540	0.0025	0.0295	0.0446	0.0029	0.0101	0.0794	0.0475	0.137	25
19	150	0.2	1.5	2185.73	0.1178	0.0126	0.0000	0.0007	0.0094	0.0532	0.1713	0.0093	0.234	12
20	150	0.2	0.8	1046.53	0.1047	0.0082	0.0122	0.0003	0.0059	0.0255	0.1927	0.0144	0.233	13
21	150	0.2	0.4	416.19	0.1027	0.0038	0.0294	0.0006	0.0001	0.0101	0.1964	0.0309	0.237	11
22	150	0.1	1.5	1112.23	0.2131	0.0036	0.0108	0.0326	0.0000	0.0271	0.0946	0.0323	0.154	24
23	150	0.1	0.8	407.26	0.1987	0.0021	0.0297	0.0278	0.0075	0.0099	0.1015	0.0558	0.167	22
24	150	0.1	0.4	154.89	0.2048	0.0011	0.0387	0.0298	0.0740	0.0038	0.0985	0.1027	0.205	16
25	150	0.05	1.5	576.25	0.4219	0.0013	0.0243	0.0756	0.0544	0.0140	0.0478	0.0929	0.155	23
26	150	0.05	0.8	373.16	0.4066	0.0009	0.0309	0.0736	0.1583	0.0091	0.0496	0.1346	0.193	20
27	150	0.05	0.4	223.11	0.4203	0.0005	0.0362	0.0754	0.7501	0.0054	0.0480	0.2530	0.306	10

Table 7: Ranking with GRA, PSI and hybrid PSI-GRA for best combination

S/N	*Process parameters*			*Performance criteria*			*Multi-criteria decision-making techniques*		
	N (rpm)	*f* (mm/rev)	*d* (mm)	*MRR* (mm^3/min)	*CEE* (kWh)	*TWR* (μm/min)	*GRA*	*PSI*	*PSI-GRA*
1	600	0.2	1.5	10311.47	0.0553	0.1255	19	1	16
2	600	0.2	0.8	4856.69	0.0433	0.0710	4	2	4
3	600	0.2	0.4	2752.23	0.0407	0.0405	7	3	5
4	600	0.1	1.5	5197.77	0.0867	0.0520	2	5	2
5	600	0.1	0.8	2851.59	0.0800	0.0273	5	8	6
6	600	0.1	0.4	1315.29	0.0707	0.0145	12	9	11
7	600	0.05	1.5	2951.95	0.1407	0.0252	8	14	9
8	600	0.05	0.8	1527.39	0.1366	0.0129	13	19	13
9	600	0.05	0.4	788.07	0.1288	0.0086	16	21	17
10	355	0.2	1.5	5846.11	0.0871	0.0415	1	4	1
11	355	0.2	0.8	3421.62	0.0771	0.0296	3	6	3
12	355	0.2	0.4	1734.50	0.0718	0.0151	10	7	8
13	355	0.1	1.5	3249.36	0.1556	0.0171	6	15	7
14	355	0.1	0.8	1852.17	0.1422	0.0107	11	17	12
15	355	0.1	0.4	851.90	0.1328	0.0053	15	18	15
16	355	0.05	1.5	1645.72	0.2747	0.0076	20	26	21
17	355	0.05	0.8	915.78	0.2604	0.0049	22	27	23
18	355	0.05	0.4	414.76	0.2540	0.0025	24	25	24
19	150	0.2	1.5	2185.73	0.1178	0.0126	9	12	10
20	150	0.2	0.8	1046.53	0.1047	0.0082	14	13	14
21	150	0.2	0.4	416.19	0.1027	0.0038	18	11	18
22	150	0.1	1.5	1112.23	0.2131	0.0036	17	24	19
23	150	0.1	0.8	407.26	0.1987	0.0021	21	22	20
24	150	0.1	0.4	154.89	0.2048	0.0011	23	16	22
25	150	0.05	1.5	576.25	0.4219	0.0013	26	23	26
26	150	0.05	0.8	373.16	0.4066	0.0009	25	20	25
27	150	0.05	0.4	223.11	0.4203	0.0005	27	10	27

The comparative analysis of the three different multi-criteria decision-making techniques of GRA, PSI and PSI-GRA have shown that each of them may provide different ranking solutions (see Figure 1).

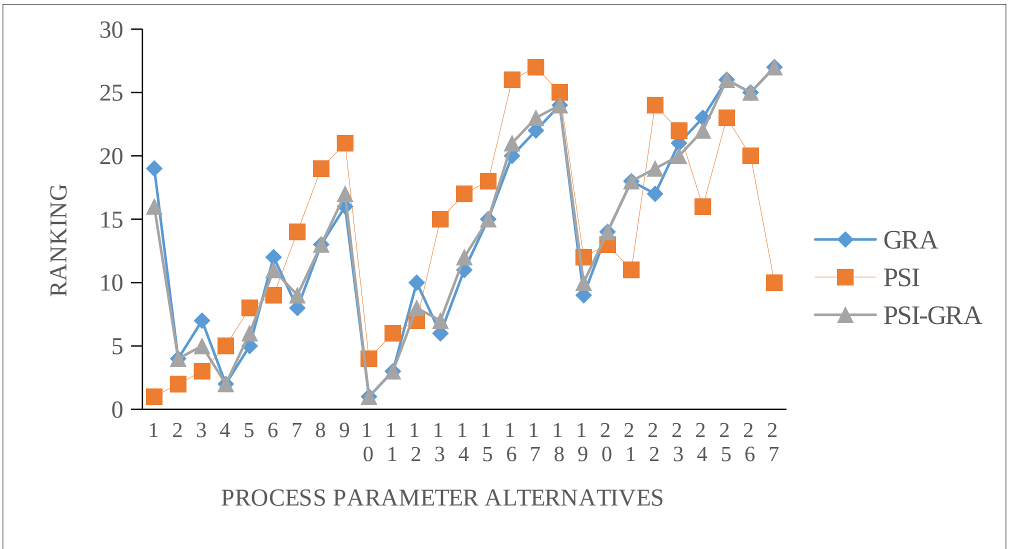

Figure 1: Graphical representation of performance ranking

Based on this observation, it is difficult to make an appropriate decision on which combination of turning process parameters can be implemented in the manufacturing shop floor. It therefore becomes essential to establish comparison metrics that will provide only one appropriate combination of optimum process parameters. There are several comparison metrics reported by Vafaei et al. (2021) including standard deviation, spearman coefficient, ranking consistency index, mean squared error, and Euclidean distance. Two comparison metrics viz. spearman coefficient (SC) and mean squared error (MSE) were taken into consideration for this study and their respective expressions are given in Equation (18) and Equation (19).

$$SC = 1 - 6\frac{\sum_{i=1}^{m}\left(x_i - x_i'\right)^2}{m\left(m^2 - 1\right)} \tag{18}$$

$$MSE = \frac{\sum_{i=1}^{m}\left(x_i - x_i'\right)^2}{m} \tag{19}$$

Where $x_i - x_i'$ is the difference between ranks obtained through sets of process parameters, and m is the number of alternatives (combinations). The rankings of spearman coefficients and mean squared errors are obtained by taking the averages of each set of techniques as presented in Table 8 and Table 9 respectively. The results show that the rankings of spearman coefficients and mean squared errors are consistently the same. It should be noted that in spearman coefficients, the highest coefficient is the best and most desirable while in mean squared errors, the lowest is highly ranked and the most desirable. The PSI-GRA is the first technique for both spearman coefficients and mean squared errors rankings followed by GRA and the last technique is PSI. Based on this ranking framework, it can be deduced that the appropriate set of multi-criteria decision-making techniques is from the hybrid PSI-GRA technique. The appropriateness of the hybrid PSI-GRA technique can be supported by the fact that it accommodates the strengths of both GRA and PSI techniques. GRA is viable and more robust when dealing with uncertainty (Kuo et al. 2008). PSI is faster and does not need to assign the relative importance of the attributes (Maniya and Bhatt, 2010). In summary, the combination of optimum turning parameters from the hybrid PSI-GRA technique can help to achieve higher performance of turning operations including lower cost, higher productivity and higher product quality.

Table 8: Ranking of Spearman coefficients

	GRA	PSI	PSI-GRA	Average	Ranking
GRA		0.645	0.717	0.681	3
PSI	0.645		0.990	0.817	2
PSI-GRA	0.717	0.990		0.853	1

Table 9: Ranking of mean squared errors

	GRA	PSI	PSI-GRA	Average	Ranking
GRA		43.111	34.370	38.741	3
PSI	43.111		1.185	22.148	2
PSI-GRA	34.370	1.185		17.778	1

Conclusion

The main objective of this study was to compare three multi-criteria decision-making techniques specifically GRA, PSI and hybrid PSI-GRA in providing optimum machining conditions including spindle speed, feed rate and depth of cut. The performance criteria were material removal rate, consumed electrical and tool wear rate for turning operations. The techniques were demonstrated using example data to provide key findings. The hybrid PSI-GRA technique provided the best combination of efficient process parameters similar to GRA method i.e., spindle speed N = 355 rpm, feed rate f = 0.05 mm/rev and depth of cut d = 1.5 mm with associated material removal rate MRR = 5846.11 mm^3/min; consumed electrical energy CEE = 0.0871 kWh; and tool wear rate TWR = 0.0415 μm/min. The comparative analysis of the three different multi-criteria decision-making techniques of GRA, PSI and PSI-GRA have shown that each of them may provide different solutions. Based on this observation, it is difficult to make an appropriate decision on which multi-criteria decision-making technique can be implemented in the manufacturing shop floor. However, through the computation of spearman coefficients and mean squared errors, it was found that the hybrid PSI-GRA technique is most appropriate compared to GRA and PSI. The combination of optimum turning parameters from the hybrid PSI-GRA technique can help to achieve higher performance of turning operations including lower cost, higher productivity and higher product quality. The proposed approach in this study can be extended for decision making in other manufacturing operations.

Declaration of Interest

The author declares that there is no conflict of interest concerning this work.

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